Journal of Biostatistics and Epidemiology

J Biostat Epidemiol. 2019;5(2): 137-147

Original Article

Truncated log-logistic Family of Distributions

Mohadese Akbarinasab, Ali Reza Arabpour, Abbas Mahdavi*

Department of Statistics, Faculty of Mathematics and Computer, Shahid Bahonar University of Kerman, Kerman, Iran.

ARTICLE INFO

Received 10.10.2018
Revised 15.11.2018
Accepted 16.01.2019
Published 01.05.2019

Key words:
Hazard rate function;
Log-logistic distribution;
Maximum-likelihood estimation;
Survival reliability function

ABSTRACT

Background & Aim: There are various data associated with any events in the world which need to be analyzed. In response to this, many researchers attempt to find appropriate methods that could better fit these data using new models. One of these methods is to introduce new distributions which could better describe available data. During last few years, new distributions have been extended based on existing well-known distributions. Usually, new distributions have more parameters than existing ones. This addition of parameter(s) has been proved useful in exploring tail properties and also for improving the goodness-of-fit of the family under study.
Methods & Materials: A new family of distributions is introduced by using truncated log-logistic distribution. Some statistical and reliability properties of the new family are derived.
Results: Four special lifetime models of the new family are investigated. We estimate the parameters by maximum likelihood method. The obtained results are validated using a real dataset and it is shown that the new distributions provide a better fit than some other known distributions.
Conclusion: We have provided four new distributions. The flexibility of the proposed distributions and increased range of skewness was able to fit and capture features in one real dataset much better than some competitor distributions

Introduction

In recent years, developing an extended class of classical distributions in areas such as survival data analysis, finance, risk modeling, insurance, modeling rare events and etc, to enhance its flexibility for better exploration of the real-life phenomenon is a common technique. So, several ways for generating new distributions from classic ones were developed. The well-known generators are the following: Beta-G distribution family that was introduced by Eugene et al. (1), McDonald class of distributions by Alexander et al. (2), gamma-G type 1 by Zografos and Balakrishnan (3) and Amini et al. (4), gamma-G type 2 by Ristić and Balakrishnan (5) and Amini et al. (4), odd exponentiated generalized by Cordeiro et al. (6), transformed-transformer (T-X) by Alzaatreh et al. (7), exponentiated T-X by Alzaghal et al. (8), odd Weibull-G by Bourguignon et al. (9), exponentiated half-logistic by Cordeiro et al. (10), T-X{Y}-quantile based approach by Aljarrah et al. (11), Lomax-G by Cordeiro et al. (12), Kumaraswamy-G class of distributions by Cordeiro et al. (13), Kumaraswamy odd log-logistic-G by Alizadeh et al. (14), logistic-X by Tahir et al. (15) and alpha power transformation family of distributions introduced by Mahdavi and Kundu (16).

Recently Mahdavi and Oliveira Silva (17) proposed a new generator of distributions, which is called truncated F-G (TF-G) family of

* Corresponding Author: a.mahdavi@vru.ac.ir.

Please cite this article in press as Akbarinasab M, Arabpour A, and Mahdavi A. Truncated log-logistic Family of Distributions. J Biostat Epidemiol. *2019; 5(2): 252-258*

distributions. They have replaced the Beta probability density function (PDF) in Beta-G family of distributions with PDF of a truncated random variable on support $(0, 1)$. Suppose $f(x, \xi_1)$ be the PDF of a continuous random variable U, which is supported on interval (a, b), where $a \leq 0, b \geq 1$ and ξ_1 are the parameters specifying the distribution. The cumulative distribution function (CDF) of the truncated random variable U on $(0, 1)$ is given by:

$$F_{U_t}(x, \xi_1) = \frac{1}{\int_0^1 f(t, \xi_1)dt} \int_0^u f(t, \xi_1)dt. \quad (1)$$

Using (1) the TF-G family of distributions has been defined by

$$F_X(x, \xi_1, \xi_2) = \frac{1}{\int_0^1 f(t, \xi_1)dt} \int_0^{G(x, \xi_2)} f(t, \xi_1)dt, \quad (2)$$

where $G(x, \xi_2)$ is CDF of any baseline distribution dependence on a parameter vector ξ_2. Mahdavi and Oliveira Silva (17) introduced truncated exponential-exponential (TEE) distribution by taking $f(x, \xi_1)$ and $G(x, \xi_2)$ as PDF and CDF of exponential distributions with means $1/\alpha$ and $1/\lambda$, respectively. The PDF of TEE distribution is given by

$$f_X(x, \alpha, \lambda) = \frac{\alpha\lambda e^{-\lambda x} e^{\alpha e^{-\lambda x}}}{e^{\alpha} - 1}, x > 0, \alpha \neq 0, \lambda > 0.$$

It is observed that TEE distribution has several interesting properties and it can be used entirely effectively to analyze lifetime data which may sometimes be a competitor to the Weibull, gamma and other two-parameter life time models.

In this paper, we use the PDF of a log-logistic distribution as $f(x, \xi_1)$ in (2) and propose truncated log-logistic-G (TLL-G) family of distributions. We provide some mathematical properties of the new family and study four special models. The present models are more flexible than the existing models because the extra parameters allow us to obtain all possible shapes of the density as well as the hazard functions. The hazard functions of this models may be monotone, non-monotone, increasing, decreasing, unimodal and bathtub shaped. Hence, TLL-G family is suitable to model and analyze a wide variety of real lifetime data applications. Moreover, as shown in the data analysis examples, the proposed sub-models provide a better fit to such data than other existing models.

Methods

Truncated log-logistic-G family

Let U be a log-logistic random variable with PDF

$$f(x, \alpha, \beta) = \frac{\alpha\beta x^{\alpha-1}}{(1+\beta x^{\alpha})^2}, x > 0, \alpha > 0, \beta > 0. \quad (3)$$

Using (3) as $f(x, \xi_1)$ in (2), we introduce the new TLL-G family of distributions for each continuous baseline $G(x, \xi_2)$ distribution as follow:

$$F_X(x, \alpha, \beta, \xi_2) = \frac{1+\beta}{G(x, \xi_2)^{-\alpha} + \beta}, \alpha > 0, \beta > 0.$$

(4) The PDF follows from (4) is

$$f_X(x, \alpha, \beta, \xi_2) = \frac{\alpha(1+\beta)g(x, \xi_2)G(x, \xi_2)^{\alpha-1}}{\{1+\beta G(x, \xi_2)^{\alpha}\}^2},$$

Proposition 1.

If $G(x, \xi_2)$ is a decreasing function, and $\alpha \leq 1$, then $f_X(x, \alpha, \beta, \xi_2)$ is a decreasing function.

Proof. By taking log $f_X(x, \alpha, \beta, \xi_2)$ and using the fact that sum of three decreasing functions is decreasing function, the result easily follows.

It is easy to see that $f_X(x,\alpha,\beta,\xi_2)$ is a weighted version of $g(x,\xi_2)$, where

$$\omega(x)=\frac{G(x,\xi_2)^{\alpha-1}}{\{1+\beta G(x,\xi_2)^{\alpha}\}^2}. \quad (5)$$

Weighted distributions have seen much use as a tool in the selection of appropriate models for observed data drawn without a proper frame. The weight function $\omega(x)$ defined in (5) can be increasing, decreasing or unimodal depending on values of α and β.

survival function

The survival function $S_X(x,\alpha,\beta,\xi_2)$ and the hazard rate function (HRF) $h_X(x,\alpha,\beta,\xi_2)$ of TLL-G family is given by

$$S_X(x,\alpha,\beta,\xi_2)=\frac{1-G(x,\xi_2)^{\alpha}}{1+\beta G(x,\xi_2)^{\alpha}},$$

and

$$h_X(x,\alpha,\beta,\xi_2)=\frac{\alpha(1+\beta)g(x,\xi_2)G(x,\xi_2)^{\alpha-1}}{\{1-G(x,\xi_2)^{\alpha}\}\{1+\beta G(x,\xi_2)^{\alpha}\}}.$$

Quantile function

The p-th quantile x_p of TLL-G family can be obtained as

$$x_p=G^{-1}\left\{\left(\frac{p}{1+\beta(1-p)}\right)^{\frac{1}{\alpha}},\xi_2\right\}. \quad (6)$$

If y_p be the p-th quantile of $G(x,\xi_2)$, then from (6) it follows that $x_p\le y_p$ for $\alpha\le 1$, and $x_p\ge y_p$ for $\alpha>1,\beta\to 0$.

It follows that for $\alpha\le 1$, $G(x,\xi_2)$ has a heavier tail than $f_X(x,\alpha,\beta,\xi_2)$, and for β close to 0, if $\alpha>1$, then it is the other way.

Results

Special Models

In this section, we provide four special cases of the TLL-G family. These special models generalize some well-known distributions in the literature of lifetime analysis. The four baseline models which are used to generate special models are exponential, Weibull, gamma and generalized exponential distribution introduced by Gupta and Kundu (18).

Truncated log –logistic -exponential distribution

We introduce the Truncated log-logistic-exponential (TLLE) distribution by taking $G(x,\xi_2)$ in (4) as CDF of an exponential distribution with mean $1/\lambda$.

Definition. 1: The non-negative random variable X has the TLLE distribution denoted by $\mathrm{TLLE}(\alpha,\beta,\lambda)$, with the shape parameters as $\alpha\neq 0$ and $\beta>0$, and scale parameter $\lambda>0$, if the CDF of X is

$$F_X(x,\alpha,\beta,\lambda)=\frac{1+\beta}{(1-e^{-\lambda x})^{-\alpha}+\beta}, x>0.$$

The PDF of TLLE distribution is given by

$$f_X(x,\alpha,\beta,\lambda)=\frac{\alpha(1+\beta)\lambda e^{-\lambda x}(1-e^{-\lambda x})^{\alpha-1}}{\{1+\beta(1-e^{-\lambda x})^{\alpha}\}^2}, x>0.$$

Figure 1 (a) shows some of the different shapes of $\mathrm{TLLE}(\alpha,\beta,\lambda)$ for selected values of the shape parameters and fixed scale parameter $\lambda=1$. It is a unimodal function if $\alpha>1$ and a decreasing function if $\alpha\le 1$.

The survival function and HRF for the TLLE distribution are given in the following forms

$$S_X(x,\alpha,\beta,\lambda)=\frac{1-(1-e^{-\lambda x})^{\alpha}}{1+\beta(1-e^{-\lambda x})^{\alpha}}, x>0,$$

and

$$h_X(x,\alpha,\beta,\lambda)=\frac{\alpha(1+\beta)\lambda\, e^{-\lambda x}(1-e^{-\lambda x})^{\alpha-1}}{\{1-(1-e^{-\lambda x})^{\alpha}\}\{1+\beta(1-e^{-\lambda x})^{\alpha}\}},$$
$x>0.$

Different shapes of the HRF are plotted in Figure 2 (a). It is exhibited by Figure 2 (a) that the HRF of the TLLE distribution can take decreasing shape if $\alpha\le 1$ and upside-down bathtub shape for $\alpha>1$. Both converge to λ for $x\to\infty$. The p-th quantile function of TLLE distribution is given by

$$x_p=\frac{-1}{\lambda}\log\left\{1-\left(\frac{p}{1+\beta(1-p)}\right)^{\frac{1}{\alpha}}\right\}.$$

It is easy to generating random numbers from TLLE distribution by using the following simple formula

$$X=\frac{-1}{\lambda}\log\left\{1-\left(\frac{U}{1+\beta(1-U)}\right)^{\frac{1}{\alpha}}\right\},$$

where U is a uniformly distributed random variable on $(0,\ 1)$ interval.

Truncated log-logistic-Weibull distribution

The Truncated log-logistic-Weibull (TLLW) distribution is defined by taking $G(x,\xi_2)$ in (4) as a CDF of Weibull distribution with following CDF:

$$G(x,\gamma,\lambda)=1-e^{-\lambda x^{\gamma}},x>0,\gamma>0,\lambda>0.$$

Definition. 2: The non-negative random variable X has the TLLW distribution denoted by $\text{TLLW}(\alpha,\beta,\gamma,\lambda)$, with the shape parameters as $\alpha\neq 0$, $\beta>0$ and $\gamma>0$, and scale parameter $\lambda>0$, if the CDF of X is

$$F_X(x,\alpha,\beta,\gamma,\lambda)=\frac{1+\beta}{(1-e^{-\lambda x^{\gamma}})^{-\alpha}+\beta},x>0.$$

(7)

The corresponding PDF to (7) is

$$f_X(x,\alpha,\beta,\gamma,\lambda)=\frac{\alpha(1+\beta)\gamma\, x^{\gamma-1}\lambda\, e^{-\lambda x^{\gamma}}(1-e^{-\lambda x^{\gamma}})^{\alpha-1}}{\{1+\beta(1-e^{-\lambda x^{\gamma}})^{\alpha}\}^2},$$
$x>0.$

Different shapes of $\text{TLLW}(\alpha,\beta,\gamma,\lambda)$ for selected values of the shape parameters and fixed scale parameter $\lambda=1$ are plotted in Figure 1 (b). It could be a unimodal function or a decreasing function depending on different values of shape parameters.

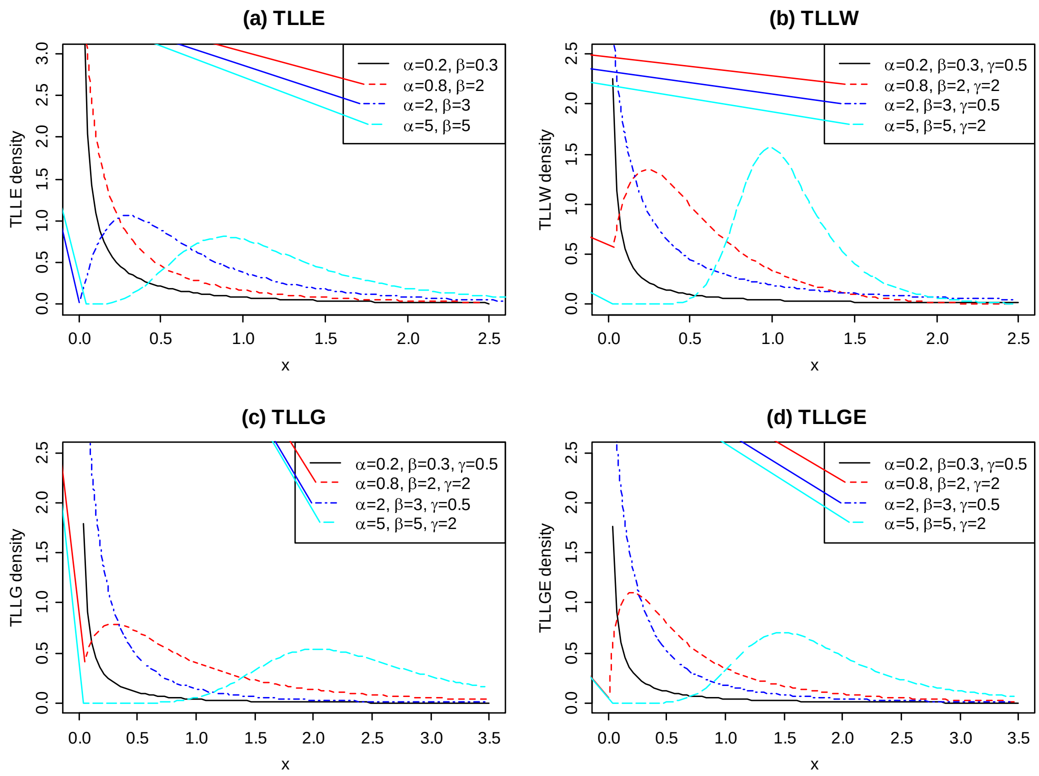

Figure 1. (a):(d) the PDFs plots for TLL-G sub-models with various shape parameters and fixed scale parameter $\lambda = 1$.

The survival function and HRF of a TLLW distributed random variable are given by

$$S_X(x,\alpha,\beta,\gamma,\lambda)=\frac{1-(1-e^{-\lambda x^{\gamma}})^{\alpha}}{1+\beta(1-e^{-\lambda x^{\gamma}})^{\alpha}}, x>0,$$

and

$$h_X(x,\alpha,\beta,\gamma,\lambda)=\frac{\alpha(1+\beta)\gamma x^{\gamma-1}\lambda e^{-\lambda x^{\gamma}}(1-e^{-\lambda x^{\gamma}})^{\alpha-1}}{\{1-(1-e^{-\lambda x^{\gamma}})^{\alpha}\}\{1+\beta(1-e^{-\lambda x^{\gamma}})^{\alpha}\}},$$

$x>0.$

Various shapes of the HRF are plotted in Figure 2 (b). It could be decreasing, increasing or first decreasing and then increasing function.

The p-th quantile function of (7) is

$$x_p=\left(\frac{-1}{\lambda}\log\left\{1-\left(\frac{p}{1+\beta(1-p)}\right)^{\frac{1}{\alpha}}\right\}\right)^{\frac{1}{\gamma}}.$$

Generating random numbers from TLLW distribution could be easily obtain by using the following formula

$$X=\left(\frac{-1}{\lambda}\log\left\{1-\left(\frac{U}{1+\beta(1-U)}\right)^{\frac{1}{\alpha}}\right\}\right)^{\frac{1}{\gamma}},$$

where U is a uniformly distributed random variable on $(0,\ 1)$ interval.

Truncated log-logistic-gamma distribution

The gamma CDF with shape parameter

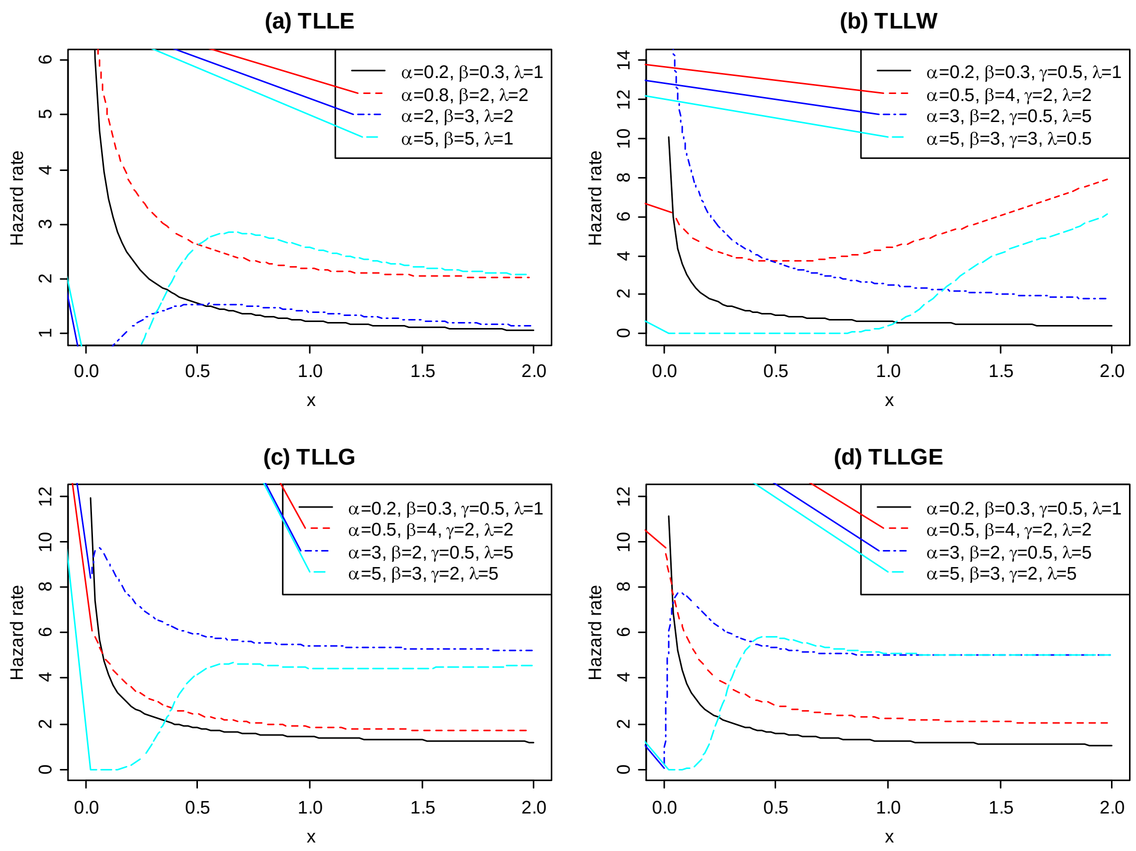

Figure 2. (a):(d) the HRFs plots for TLL-G sub-models with various parameters and fixed scale parameter $\lambda=1$.

$\gamma>0$ and scale parameter $\lambda>0$ is $G(x,\gamma,\lambda)=\Gamma_{\lambda x}(\gamma)/\Gamma(\gamma), x>0,$ where $\Gamma(\gamma)=\int_0^\infty t^{\gamma-1}e^{-t}dt$ is gamma function and $\Gamma_x(\gamma)=\int_0^x t^{\gamma-1}e^{-t}dt$ is the incomplete gamma function. The Truncated log-logistic-gamma (TLLG) distribution by taking $G(x,\xi_2)$ in (4) as CDF of gamma distribution is defined as follows:

Definition. 3: The non-negative random variable X has the TLLG distribution denoted by $\text{TLLG}(\alpha,\beta,\gamma,\lambda)$, with the shape parameters as $\alpha\neq 0$, $\beta>0$, γ and scale parameter $\lambda>0$, if the CDF of X is

$$F_X(x,\alpha,\beta,\gamma,\lambda)=\frac{1+\beta}{\Gamma(\gamma)^{\alpha}\Gamma_{\lambda x}(\gamma)^{-\alpha}+\beta}, x>0.$$

The associated PDF reduces to

$$f_X(x,\alpha,\beta,\gamma,\lambda)=\frac{\alpha\,(1+\beta)\lambda^{\gamma}x^{\gamma-1}e^{-\lambda x}\Gamma_{\lambda x}(\gamma)^{\alpha-1}}{\Gamma(\gamma)^{\alpha}\{1+\beta\,\Gamma(\gamma)^{-\alpha}\Gamma_{\lambda x}(\gamma)^{\alpha}\}^2},$$

$$x>0. \quad (8)$$

Plots of the density function (8) for selected parameter values are given in Figure 1 (c). It could be a unimodal function or a decreasing function depending on values of the shape parameters.

The survival function and HRF for the TLLG distribution are given in the following forms

$$S_X(x,\alpha,\beta,\gamma,\lambda)=\frac{1-\Gamma(\gamma)^{-\alpha}\Gamma_{\lambda x}(\gamma)^{\alpha}}{1+\beta\Gamma(\gamma)^{-\alpha}\Gamma_{\lambda x}(\gamma)^{\alpha}},x>0,$$

and

$$h_X(x,\alpha,\beta,\gamma,\lambda)=\left(\frac{\alpha\,(1+\beta)\lambda^{\gamma}x^{\gamma-1}e^{-\lambda x}\Gamma_{\lambda x}(\gamma)^{\alpha-1}}{\Gamma(\gamma)^{\alpha}}\right)$$
$$\left(\frac{1}{\{1-\Gamma(\gamma)^{-\alpha}\Gamma_{\lambda x}(\gamma)^{\alpha}\}\{1+\beta\,\Gamma(\gamma)^{-\alpha}\Gamma_{\lambda x}(\gamma)^{\alpha}\}}\right),x>0.$$

The plots of the HRF are plotted in Figure 2 (c). It could take decreasing shape and upside-down bathtub shape depending on values of the shape parameters.

Truncated log-logistic-generalized exponential distribution

The CDF of generalized exponential distribution with parameters γ and λ is $G(x,\gamma,\lambda)=(1-e^{-\lambda x})^{\gamma},x>0$. We introduce the Truncated log-logistic-generalized exponential (TLLGE) distribution by taking $G(x,\xi_2)$ in (4) as a CDF of generalized exponential distribution.

Definition. 4: The non-negative random variable X has the TLLGE distribution denoted by $\text{TLLGE}(\alpha,\beta,\gamma,\lambda)$, with the shape parameters as $\alpha\neq0$, $\beta>0$ and γ, and scale parameter $\lambda>0$, if the CDF of X is

$$F_X(x,\alpha,\beta,\gamma,\lambda)=\frac{1+\beta}{(1-e^{-\lambda x})^{-\alpha\gamma}+\beta},x>0,$$

and the PDF of TLLGE distribution reduces to

$$f_X(x,\alpha,\beta,\gamma,\lambda)=\frac{\alpha\gamma(1+\beta)\lambda\,e^{-\lambda x}(1-e^{-\lambda x})^{\alpha\gamma-1}}{\{1+\beta(1-e^{-\lambda x})^{\alpha\gamma}\}^{2}},$$
$$x>0.$$

The graphs of the TLLGE density in Figure 1 (d) show that the function could be unimodal or decreasing for the various values of the shapes parameters.

The survival function and HRF for the TLLGE distribution

are given in the following forms

$$S_X(x,\alpha,\beta,\gamma,\lambda)=\frac{1-(1-e^{-\lambda x})^{\alpha\gamma}}{1+\beta(1-e^{-\lambda x})^{\alpha\gamma}},x>0,$$

and

$$h_X(x,\alpha,\beta,\gamma,\lambda)=\frac{\alpha\gamma(1+\beta)\lambda e^{-\lambda x}(1-e^{-\lambda x})^{\alpha\gamma-1}}{\{1-(1-e^{-\lambda x})^{\alpha\gamma}\}\{1+\beta(1-e^{-\lambda x})^{\alpha\gamma}\}},$$
$$x>0.$$

Plots of the HRF for TLLGE distribution are displayed in Figure 2 (d). It is depicted by Figure 2 (d) that the HRF of the TLLGE distribution can take decreasing shape and upside-down bathtub shape. Both converge to λ for $x\to\infty$.

The p-th quantile function of TLLGE distribution is given by

$$x_p=\frac{-1}{\lambda}\log\left\{1-\left(\frac{p}{1+\beta(1-p)}\right)^{\frac{1}{\alpha\gamma}}\right\}.$$

It is easy to generating random numbers from TLLGE distribution by using the following simple formula:

$$X=\frac{-1}{\lambda}\log\left\{1-\left(\frac{U}{1+\beta(1-U)}\right)^{\frac{1}{\alpha\gamma}}\right\},$$

where U is a uniformly distributed random variable on $(0,\,1)$ interval.

Discussion

Maximum Likelihood Estimation

We now determine the maximum likelihood estimates (MLEs) of the parameters of the TLL-G family of distributions from complete samples only.

Let $x_1, x_2, ..., x_n$ be a random sample from TLL-G distributions with unknown parameters $\boldsymbol{\Theta} = (\alpha, \beta, \xi_2)^T$, an $k \times 1$ parameter vector. Then, the log-likelihood function based on the given random sample is

$$\ell(\boldsymbol{\Theta}) = n \log \alpha + n \log(1+\beta) + \sum_{i=1}^{n} \log\, g(x_i, \xi_2) +$$
$$(\alpha - 1)\sum_{i=1}^{n} \log G(x_i, \xi_2) - 2\sum_{i=1}^{n} \log\{1 + \beta\, G(x_i, \xi_2)^{\alpha}\}.$$

The first order derivatives of $\ell(\boldsymbol{\Theta})$ are

$$\frac{\partial \ell(\boldsymbol{\Theta})}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \log G(x_i, \xi_2)$$
$$-2\beta \sum_{i=1}^{n} \frac{G(x_i, \xi_2)^{\alpha} \log G(x_i, \xi_2)}{1 + \beta G(x_i, \xi_2)^{\alpha}} = 0,$$
$$\frac{\partial \ell(\boldsymbol{\Theta})}{\partial \beta} = \frac{n}{1+\beta} - 2\sum_{i=1}^{n} \frac{G(x_i, \xi_2)^{\alpha}}{1 + \beta G(x_i, \xi_2)^{\alpha}} = 0,$$
$$\frac{\partial \ell(\boldsymbol{\Theta})}{\partial \xi_2} = \sum_{i=1}^{n} \frac{g^{(\xi_2)}(x_i, \xi_2)}{g(x_i, \xi_2)} + (\alpha - 1)\sum_{i=1}^{n} \frac{G^{(\xi_2)}(x_i, \xi_2)}{G(x_i, \xi_2)}$$
$$-2\beta \sum_{i=1}^{n} \frac{G^{(\xi_2)}(x_i, \xi_2)^{\alpha - 1}}{1 + \beta G(x_i, \xi_2)^{\alpha}} = 0,$$

where $h^{(\xi)}(.)$ denotes the derivative of h respect to ξ. These equations cannot be solved analytically because of their nonlinear structure. Mathematical and statistical software can be used to solve them numerically using iterative methods such as the Newton-Raphson type algorithms. In this paper, we have used the "mle" function that is under the "stats4" package could offer numerical solution to such problems in R, R-language (19).

Let $\boldsymbol{\Theta} = (\hat{\alpha}, \hat{\beta}, \xi_2)^T$ denotes the MLE of $\boldsymbol{\Theta} = (\alpha, \beta, \xi_2)^T$. The distribution of $\boldsymbol{\Theta}$ under regularity conditions as $n \to \infty$ is k -variate normal distribution with mean $\boldsymbol{\Theta}$ and covariance given by the inverse of expected information matrix. This asymptotic behavior is valid if the expected information matrix is replaced by the observed information matrix. The asymptotic obtained multivariate normal distribution can be used to construct approximate confidence intervals for the individual parameters and for the hazard rate and survival functions. Package "numDeriv" of R language can be used to compute the Hessian matrix and its inverse, standard errors and asymptotic confidence intervals.

Application: Guinea Pigs Dataset

In this section, we use an uncensored data set corresponding to survival times of 72 guinea pigs injected with different amount of tubercle and was studied by Bjerkedal et al. (20). The data represents the survival times of Guinea pigs in days. The data are given below:

12 15 22 24 24 32 32 33 34 38 38 43 44 48 52 53 54 54 55 56 57 58 58 59 60 60 60 60 61 62 63 65 65 67 68 70 70 72 73 75 76 76 81 83 84 85 87 91 95 96 98 99 109 110 121 127 129

131 143 146 146 175 175 211 233 258 258 263 297 341 341 376.

Many authors have been used this dataset to investigate their proposed models. For example, Gupta and Kundu (21) proposed weighted exponential distribution (WE) and compared it with Weibull, generalized exponential, and gamma distributions based on this dataset, and they observed that the WE was better than all of them.

Now, we compare the four proposed sub-models of TLL-G family with

TEE distribution described in Section 1, Topp-Leone generalized exponential

WE distribution with PDF

$$f(x,\alpha,\lambda)=\frac{\alpha+1}{\alpha}\lambda e^{-\lambda x}(1-e^{-\alpha\lambda x}),$$

Gamma exponentiated-exponential (GEE) distribution Ristić and Balakrishnan (5) with PDF

$$f(x,\alpha,\theta,\lambda)=\frac{\alpha\theta}{\Gamma(\lambda)}(1-e^{-\theta x})^{\alpha-1}$$

$$[-\alpha\log(1-e^{-\theta x})]^{\lambda-1},$$

where $\alpha>0,\beta>0,\theta>0,\lambda>0,x>0.$

To see which one of these models is more appropriate to fit data the MLEs of parameters, Akaike Information Criterion (AIC) value, Bayesian Information Criterion (BIC) value, Kolmogorov-Smirnov (K-S) statistic and its associated p-value are obtained.

Table 1. The MLEs, AICs, BICs, K-S statistics and its p-values for fitted distributions.

The model	MLEs of the parameters	AIC	BIC	K-S statistic	p-value
TLLE	$\hat{\alpha}=3.2406,\hat{\beta}=6.0610,\hat{\lambda}=0.0099$	784.3102	791.1402	0.0847	0.6801
TLLW	$\hat{\alpha}=4.0362,\hat{\beta}=7.4171,\hat{\gamma}=0.8725,\hat{\lambda}=0.0198$	786.1017	795.2083	0.0888	0.6218
TLLG	$\hat{\alpha}=3.1865,\hat{\beta}=3.8155,\hat{\gamma}=1.0224,\hat{\lambda}=0.0116$	786.8936	796.0002	0.0850	0.6753
TLLGE	$\hat{\alpha}=1.8073,\hat{\beta}=6.7208,\hat{\gamma}=1.8073,\hat{\lambda}=0.0097$	786.2547	795.3614	0.0870	0.6474
TEE	$\hat{\alpha}=-3.3270,\hat{\lambda}=0.0192$	797.1394	801.6927	0.14094	0.1144
WE	$\hat{\alpha}=1.6232,\hat{\lambda}=0.0138$	791.1382	795.6915	0.1173	0.2748
GEE	$\hat{\alpha}=2.6006,\hat{\theta}=0.0083,\hat{\lambda}=2.1138$	793.2470	800.077	0.1347	0.1467

The result is given in Table 1. The TLLE distribution gives the smallest AIC, smallest BIC, smallest K-S statistic and the largest p-value. Also the TLLW, TLLG and TLLGE provide better fit to the data respect to competitor models. The histogram of the data and the plots of the fitted PDFs for the proposed special models are shown in Figure 3. Figure 4 shows the empirical and fitted survival functions for these special models.

Conflict of Interests

Authors have no conflict of interests.

Conclusion

Mahdavi and Oliveira Silva (17) show that it could use other distributions instead of beta distribution as the generator in the beta method to derive different classes of distributions. In this paper, we used truncated log-logistic distribution on support $(0,1)$ as the generator and introduced new family of distributions called truncated log-logistic-G (TLL-G) family of distributions. Various properties of the new family are obtained. The estimation of parameters is approached by the method of maximum likelihood Finally, future works is needed to implement and investigate the behavior of the proposed family to expand other family of distributions.

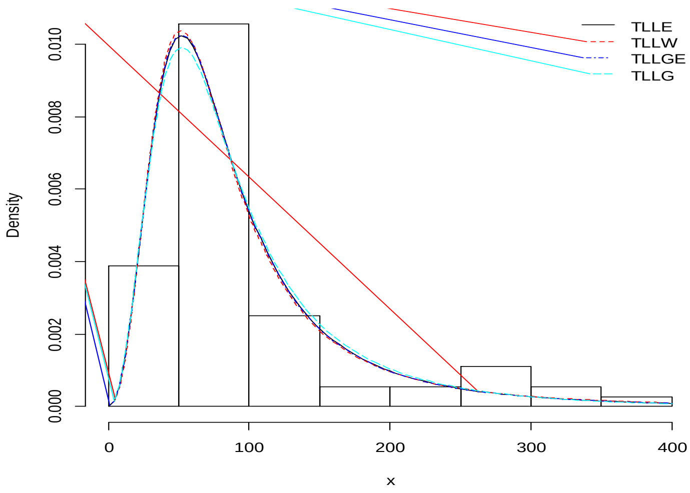

Figure 3. Fitted PDFs for the TLLE, TLLW, TLLG and TLLGE models to the guinea pigs data.

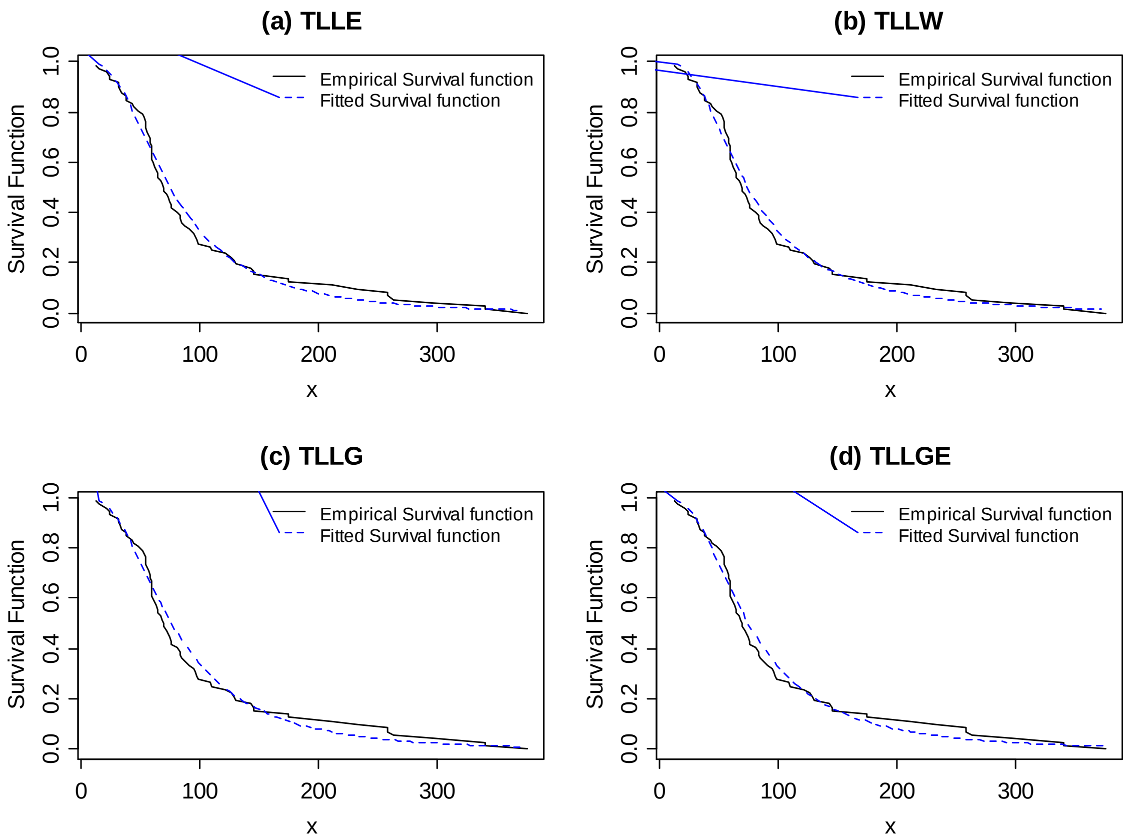

Figure 4. (a):(d) the empirical survival functions and fitted survival functions for the TLLE, TLLW, TLLG and TLLGE models.

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