Appendix. A.

In this appendix, we explain the structure of full likelihood

$$L(O\mid\omega_i)=\prod\int L(O_i\mid\omega_i)\,g(\omega_i)d\omega_i$$, where

$L_i\,(\lambda_0(.), h_0(.), \beta, \beta^*, \alpha, \theta) = L_i\,(O\mid$ for subject i and ($j = 1, 2, ..., n_i$) such as, $\delta_{(i,\,n_i+1)} = 0$.

1) we calculated the conditional likelihood for the patients who do not experience the occurrence of disease and observing death (ni = 0, $\Delta_i = 1$), we have:

$$S_i^R(t_i\mid\omega_i)=\exp(-\omega_i\sum_{j=1}^{n_i+1}\int_{T_{i(j-1)}}^{T_{ij}}Y_i(t)\lambda_i(t)dt),$$

$$S_i^D(t_i^*\mid\omega_i)=\exp(-\omega_i\int_0^{T_i^*}Y_i(t)h_i(t)dt)$$

$$L_{i1}=(1-p_i)S_i^R(\mathrm{t}_i\mid\omega_i,\mathrm{Y}_i=1)\times h_i(\mathrm{t}_i\mid\omega_i,\mathrm{Y}_i=1)^{\Delta_i}\times S^D{}_i(\mathrm{t}_i\mid\omega_i,\mathrm{Y}_i=1),$$

1) We consider $\omega_i : \; G(\frac{1}{\theta},\frac{1}{\theta})$ with probability density

$$f(\omega)=\frac{\omega^{(1/\theta-1)}\exp(-\omega/\theta)}{\Gamma(1/\theta)\theta^{(\frac{1}{\theta})}}$$

The contribution of marginal likelihood is obtained by integrating out the random effect (ω_i).

$$L_{i1}(\mathrm{O}\mid\omega_i)=\frac{(1-\mathrm{p}_i)\times(\mathrm{h}_i(t_i^*))^{\Delta_i}}{\theta^{\frac{1}{\theta}}\Gamma(\frac{1}{\theta})}\times\int_0^{\infty}(\omega_i^{(\Delta_i+\frac{1}{\theta}-1)}\exp(-\omega_i\sum_{j=1}^{n_i+1}\int_{T_{i(j-1)}}^{T_{ij}}Y_i(\mathrm{t})\lambda_i(\mathrm{t})\,\mathrm{dt}-\omega_i\int_0^{T_i^*}Y_i(\mathrm{t})\,\mathrm{h}_i(\mathrm{t})\,\mathrm{dt})-\frac{\omega_i}{\theta})d\omega_i$$

In situation 2, we have subjects that experience the recurrent event $n_i > 0$ and observing death $\Delta_i = 1$. The contribution of marginal likelihood for individual *i* can write:

$$L_{i2}(\mathrm{O}\mid\omega_i)=\frac{(1-p_i)\times\prod_{j=1}^{n_i+1}\lambda(t_{ij})^{\delta_{ij}}\times(\mathrm{h}_i(t_i^*))^{\Delta_i}}{\theta^{(1/\theta)}\Gamma(1/\theta)}\times\int_0^{\infty}(\omega^{(\Delta_i+n_i+\frac{1}{\theta}-1)}\times\exp(-\omega_i\sum_{j=1}^{n_i+1}\int_{T_{i(j-1)}}^{T_{ij}}Y_i(t)\lambda_i(\mathrm{t})\,\mathrm{dt}-\omega_i\int_0^{\tau_i}Y_i(t)\,\mathrm{h}_i(t)dt)-\frac{\omega_i}{\theta})d\omega_i$$

In situation 3, we have subjects that experience the recurrent event $n_i > 0$ but no observing death $\Delta_i = 0$. the contribution of marginal likelihood for individual *i* can write:

$$L_{i3}(\mathrm{O}) = \frac{(1-p_i)\times\prod_{j=1}^{n_i+1}\lambda(t_{ij})^{\delta_{ij}}}{\theta^{(1/\theta)}\Gamma(1/\theta)} \times \int_0^\infty (\omega^{(n_i+\frac{1}{\theta}-1)} \times \exp(-\omega_i \sum_{j=1}^{n_i+1}\int_{T_{i(j-1)}}^{T_{ij}} Y_i(t)\lambda_i(\mathrm{t})\,\mathrm{dt} - \omega_i \int_0^{T_i^*} Y_i(t)\mathrm{h}_i(t)dt) - \frac{\omega_i}{\theta})d\omega_i$$

In situation 4, we have subjects that experience neither recurrence nor death from the disease ($n_i = 0, \Delta_i = 0$).

$$L_{i4}(\mathrm{O}) = \frac{\mathrm{p}_i + (1-p_i)}{\theta^{(1/\theta)}\Gamma(1/\theta)} \times \int_0^\infty (\omega^{(\frac{1}{\theta}-1)} \times \exp(-\omega_i \sum_{j=1}^{n_i+1}\int_{T_{i(j-1)}}^{T_{ij}} Y_i(t)\lambda_i(\mathrm{t})\,\mathrm{dt} - \omega_i \int_0^{T_i^*} Y_i(t)\mathrm{h}_i(t)dt) - \frac{\omega_i}{\theta})d\omega_i$$

The marginal likelihood calculated by multiplying of the four marginal contribution of likelihood for subject *i* as follow

$$L(O_i \mid \omega_i) = L_{i1}^{I(\mathrm{n}_i=0,\Delta_i=1)} L_{i2}^{I(\mathrm{n}_i=1,\Delta_i=1)} L_{i3}^{I(\mathrm{n}_i=1,\Delta_i=0)} L_{i4}^{I(\mathrm{n}_i=0,\Delta_i=0)}$$

We can obtain full log likelihood by sum of the four marginal contribution of log-likelihood for subject i as follow

$$l(O) = \sum_{i=1}^{4} l_i(O)$$

We can obtained the log-likelihood for gap times with replace T_{ij} by S_{ij} and $\int_{T_{i(j-1)}}^{T_{ij}}$ by $\int_0^{S_{ij}}$ in expression of log-likelihood.