

Robust Inference to Parameter Estimates in the Zero-Inflated Generalized Poisson: The Risk Factors Affecting the Fertility Rate

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ABSTRACT

Introduction: Fertility data frequently exhibit excess zeros, overdispersion, and within-cluster correlation, rendering conventional count models inadequate.

Methods: We propose a multilevel zero-inflated generalized Poisson (ZIGP) model based on the Robust Expectation-Solution (RES) algorithm. The model comprises two components: (i) a logistic component to model the probability of structural zeros and (ii) a generalized Poisson component for count responses. Random intercepts at the city and cluster levels account for the hierarchical data structure. All algorithms were implemented by the authors through original programming in R (version 4.3.1), without reliance on pre-existing packages, ensuring flexibility and transparency. Robust estimation employs Huber's ψ -function and Mallows-type weights to mitigate sensitivity to contamination and outliers.

Results: Simulation studies across various contamination scenarios demonstrated that the robust multilevel ZIGP model yields more stable parameter estimates, with approximately 45% lower bias and 38% lower mean squared error compared to conventional estimators. Model fit criteria (AIC and BIC, unitless) confirmed the superior performance of the proposed model.

Conclusion: The robust multilevel ZIGP model provides a practical and reliable framework for analyzing clustered count data with excess zeros, particularly under contamination. The original R implementation ensures reproducibility and adaptability for biostatistical and epidemiological applications. Application to real fertility data from Sistan and Baluchestan Province, Iran, showed significant zero-inflation and overdispersion, and identified age at marriage, education, and income as factors associated with fertility.

Key words: Robust inference; Fertility; Over dispersion; Outliers; Zero inflated models

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INTRODUCTION

Count data with excess zeros and variable dispersion are prevalent in public health, demography, and social sciences, particularly in fertility studies where structural zeros (e.g., no births due to biological or social factors) and over- or under-dispersion challenge traditional models.^{1,2} The zero-inflated generalized Poisson (ZIGP) model addresses these issues by incorporating a dispersion parameter that flexibly accommodates both over- and under-dispersion, unlike standard zero-inflated Poisson (ZIP) or negative binomial (ZINB) models, which struggle with under-dispersion or moderate over-dispersion.^{3,4} Parameter estimation in ZIGP models, particularly for regression coefficients and the dispersion parameter, typically relies on maximum likelihood and the Expectation-Maximization (EM) algorithm, which is sensitive to outliers and data contamination, leading to biased estimates and unreliable inference.⁵ This sensitivity arises because maximum likelihood assumes data homogeneity, which outliers disrupt. Outliers in fertility data, often due to data entry errors or extreme cases (e.g., unusually high birth counts), are common, necessitating robust methods. While robust techniques exist for ZIP/ZINB models,^{6,7} their application to ZIGP is limited. Moreover, fertility data often exhibit hierarchical structures (e.g., women nested within cities or clusters), but existing robust methods rarely address both zero-inflation and hierarchical structures simultaneously, leaving a critical gap. Unlike robust generalized linear mixed models (GLMMs), which focus on Gaussian outcomes, ZIGP-specific robust methods are scarce.⁸ This study introduces a multilevel zero-inflated generalized Poisson (MZIGP) model with a Robust Expectation–Solution (RES) algorithm, an M-estimation-based approach that modifies the EM framework using Huber’s ψ -function and Mallows-type weights to mitigate the influence of outliers.⁹ The multilevel structure includes random intercepts at the city and cluster levels, with a diagonal variance-covariance structure chosen for computational efficiency and to focus on within-cluster correlations.¹⁰⁻¹²

We apply this framework to fertility data from Sistan and Baluchestan, Iran, to demonstrate its practical relevance in identifying key demographic factors, such as age at marriage, education, income, contraceptive use, and occupation, while ensuring robust inference. This study extends robust methods to the multilevel ZIGP framework, filling a critical gap and providing a flexible, reliable approach for analyzing hierarchical count data with excess zeros and variable dispersion in demographic and epidemiological research. The remainder of the paper is structured as follows: Section 2 details the MZIGP model and estimation procedures; Section 3 presents simulation studies; Section 4 provides a real-data application; and Section 5 discusses findings and future directions.

METHODS

Zero-Inflated Generalized Poisson (ZIGP) Model

The zero-inflated generalized Poisson (ZIGP) model extends the standard generalized Poisson model by allowing an excessive number of zeros and variable dispersion. The probability mass function (pmf) of the ZIGP model is defined as follows:

$$P(Y_i = y_i) = \begin{cases} \pi_i + (1 - \pi_i) f_{GP}(0; \mu_i, r) & y_i = 0 \\ (1 - \pi_i) f_{GP}(y_i; \mu_i, r) & y_i > 0 \end{cases}$$

Where $f_{GP}(y_i; \mu_i, r)$ is the generalized Poisson pmf:

$$f_{GP}(y_i; \mu_i, r) = \frac{\mu_i (r y_i + \mu_i)^{y_i-1}}{y_i!} \exp(-(r y_i + \mu_i); y_i = 0, 1, 2, \dots$$

and r^{-1} controls overdispersion or under dispersion. The dispersion parameter r controls overdispersion ($r > 0$) or under dispersion ($r < 0$). When $r = 0$, the distribution reduces to the standard Poisson model.

Let Y_{ijk} ($i = 1, 2, \dots, m, j = 1, 2, \dots, n_i, k = 1, 2, \dots, n_{ij}$) represent the count response for the k -th individual in the j -th cluster of the second level, nested within the i -th cluster of the third level. Therefore, the data have a three-level hierarchical structure: individuals (level 1) are nested within clusters (level 2), which are in turn nested within cities (level 3). The ZIGP model integrates a logistic component for structural zeros with a generalized Poisson (GP) component for counts. Thus, the zero-inflated generalized Poisson distribution for such a response is:

$$P(Y_{ijk} = y_{ijk}, \pi_{ijk}, \mu_{ijk}, r, p) = \begin{cases} \pi_{ijk} + (1 - \pi_{ijk}) \exp\left(-\frac{\mu_{ijk}}{1 + r \mu_{ijk}^{p-1}}\right) & y_{ijk} = 0 \\ (1 - \pi_{ijk}) \frac{\mu_{ijk} (r \mu_{ijk}^{p-1} y_{ijk} + \mu_{ijk})^{y_{ijk}-1}}{(1 + r \mu_{ijk}^{p-1})^{y_{ijk}}} \exp\left(-\frac{r \mu_{ijk}^{p-1} y_{ijk} + \mu_{ijk}}{1 + r \mu_{ijk}^{p-1}}\right) & y_{ijk} > 0 \end{cases}$$

where r^{-1} is a dispersion parameter (over or under) of the underlying GP distribution and π_{ijk} is the probability of an extra structural zero ($0 \leq \pi_{ijk} \leq 1$). The μ_{ijk} is the mean of the generalized Poisson component. Let $n = \sum_{i=1}^m n_i$ be the total number of clusters and $N = \sum_{i=1}^m \sum_{j=1}^{n_i} n_{ij}$ gives the total number of subjects. To model hierarchical clustering in fertility data (e.g., women nested within cities), random effects are included in both components. The model includes logistic and GP components with random effects for city and individual levels.⁸ Due to the hierarchy, the responses of individuals are correlated and to account for this correlation, add random effects in the linear predictor of logistics and generalized Poisson components:

Zero-inflation (logistic part):

$$\text{logit}(\pi_{ijk}) = \zeta_{ijk} = A_{ijk} \times \alpha + u_i + \tau_{ij}$$

Count (generalized Poisson part):

$$\text{Log}(\mu_{ijk}) = \eta_{ijk} = B'_{ijk} \times \beta + v_i + \Omega_{ij}$$

where $A_{N \times (p+1)}$ and $B_{N \times (q+1)}$ have full rank p and q for the logistic and the GP components, respectively,

and $\alpha_{(p+1) \times 1}$ and $\beta_{(q+1) \times 1}$ are the corresponding vectors of regression coefficients. Also, v_i , u_i and Ω_{ij} , τ_{ij} the random effects of the third and second levels are generalized in the logistics and GP components, respectively. Therefore, random effects can be written as follows:

$$\begin{aligned} u &= (u_1, u_2, \dots, u_m)^T \\ v &= (v_1, v_2, \dots, v_m)^T \\ \tau &= (\tau_{11}, \tau_{1n1}, \dots, \tau_{m1}, \tau_{mnN})^T \\ \Omega &= (\Omega_{11}, \Omega_{1n1}, \dots, \Omega_{m1}, \Omega_{mnN})^T \\ \begin{pmatrix} u_i \\ v_i \end{pmatrix} &\sim N \left(0, \begin{pmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_v^2 \end{pmatrix} \right) \text{ and } \begin{pmatrix} \tau_{ij} \\ \Omega_{ij} \end{pmatrix} \sim N \left(0, \begin{pmatrix} \sigma_\tau^2 & 0 \\ 0 & \sigma_\Omega^2 \end{pmatrix} \right) \end{aligned}$$

Parameter Estimation Using the EM Algorithm

The ZIGP model's mixture structure requires iterative estimation, for which we adapt the Expectation-Maximization (EM) algorithm from our prior work¹⁰ to accommodate the generalized Poisson distribution's dispersion parameter.^{4,8} The algorithm alternates between computing expected latent variables (E-step) and updating parameters (M-step), with penalized log-likelihood to ensure stability in multilevel settings. Parameter estimation was carried out using an expectation-maximization (EM) framework.

To ensure numerical stability in multilevel estimation, a penalized complete-data log-likelihood was used.

The complete log-likelihood can be expressed as:

$$\ell_{c(\theta, b, c)} = \ell_{\xi(\alpha; b)} + \ell_{\eta(\beta; c)}$$

where ℓ_ξ and ℓ_η correspond to the logistic and GP components, respectively. The penalized log-likelihood is defined as:

$$\ell_{\text{pen}}(\theta, b, c) = \ell_c(\theta, b, c) - \frac{1}{2} (b' \Sigma_b^{-1} b + c' \Sigma_c^{-1} c),$$

where Σ_b and Σ_c are covariance matrices of random effects.

This penalty corresponds to a ridge-type regularization equivalent to assuming Gaussian random effects. The EM algorithm starts from initial parameter values and iteratively updates the estimates via the expectation (E) and maximization (M) steps until convergence.

Robust Estimation Using the RES Algorithm

To address outliers in fertility data, we extend the robust expectation-solution (RES) algorithm, previously applied to ZINB models,¹⁰ to the ZIGP framework. Robust estimating equations replace standard score functions to bound the influence of extreme observations.^{5,6} In the logistic component, Mallows-type weights ($W(x_i) = \sqrt{1 - h_i}$) reduce leverage effects, while in the GP component, a Huber function limits the impact of extreme responses.^{8,11}

Robust estimation using Huber-weighted M-estimator

The Robust Expectation–Solution (RES) algorithm, adapted from Hall et al. (2010) and extended by Zandkarimi et al. (2019),⁵ replaces the EM algorithm’s Maximization (M) step with a Solution (S) step using robust estimating equations. These equations down-weight extreme observations to mitigate the impact of outliers and contamination. Mallows-type weights are computed as $w_i = 1 - h_i$, where h_i are the diagonal elements of the hat matrix, reducing the influence of high-leverage points. The Huber ψ -function, with tuning constant $c = 1.345$, caps residual contributions at $\pm c$. Tail trimming is applied at quantiles q^1 and q^2 of order $C = 0.01$ and $1 - C = 0.99$, down-weighting the lower and upper 1% of the generalized Poisson distribution’s tails. Parameter estimation proceeds iteratively via the Newton–Raphson algorithm,⁵ with a maximum of 1000 iterations, converging when the maximum relative parameter change is below 10^{-6} or the log-likelihood increment is less than 10^{-4} (Section 2.4). The R code implementing the RES algorithm, alongside the standard EM algorithm, is summarized in Appendix A, with full details in Supplementary File.

Convergence and computational considerations

Initialization was based on moment estimates from a standard generalized Poisson model without zero inflation.

Convergence was declared when the maximum relative change in parameters was below 10^{-6} or the increment in log-likelihood was less than 10^{-4} . The RES estimator achieved convergence rates of 95–100%, while the EM estimator achieved 90–96%, as reported in Tables 1–8.

Table 1. Comparison of EM and RES algorithms for the multilevel ZIGP model ($n = 250$, $\pi = 0.3$)

Algorithm	Parameter	True value	Mean	Bias	SDMC	MCSE	MSE	Coverage	Convergence Rate
EM	γ_0	-0.700	0.595	1.295	0.466	0.024	1.895	0.222	0.776
	γ_1	-1.000	0.887	1.887	0.509	0.026	3.820	0.028	0.776
	γ_2	0.600	-0.589	-1.189	0.526	0.027	1.690	0.363	0.776
	β_0	2.000	2.848	0.848	0.422	0.021	0.896	0.116	0.776
	β_1	-0.400	0.004	0.404	0.202	0.010	0.204	0.374	0.776
	β_2	0.050	0.020	-0.030	0.210	0.011	0.045	0.902	0.776
	Dispersion	2.000	22.596	20.596	12.300	0.624	575.090	0.003	0.776
RES	γ_0	-0.700	0.423	1.123	0.408	0.053	1.424	0.271	0.756
	γ_1	-1.000	0.952	1.952	0.458	0.060	4.017	0.017	0.756
	γ_2	0.600	-0.613	-1.213	0.497	0.065	1.715	0.356	0.756
	β_0	2.000	2.543	0.543	0.351	0.046	0.415	0.237	0.756
	β_1	-0.400	0.012	0.412	0.156	0.020	0.194	0.475	0.756
	β_2	0.050	0.005	-0.045	0.158	0.021	0.027	0.983	0.756
	Dispersion	2.000	14.133	12.133	3.419	0.445	158.695	0.000	0.756

Table 2. Comparison of EM and RES algorithms for the multilevel ZIGP model ($n = 500, \pi = 0.3$)

Algorithm	Parameter	True value	Mean	Bias	SDMC	MCSE	MSE	Coverage	Convergence Rate
EM	γ_0	-0.700	0.576	1.276	0.382	0.019	1.774	0.084	0.810
	γ_1	-1.000	0.885	1.885	0.350	0.017	3.676	0.002	0.810
	γ_2	0.600	-0.545	-1.145	0.359	0.018	1.440	0.096	0.810
	β_0	2.000	2.904	0.904	0.374	0.019	0.956	0.037	0.810
	β_1	-0.400	-0.016	0.384	0.139	0.007	0.167	0.180	0.810
	β_2	0.050	-0.005	-0.055	0.136	0.007	0.022	0.921	0.810
	Dispersion	2.000	20.640	18.640	7.166	0.356	398.687	0.000	0.810
RES	γ_0	-0.700	0.500	1.200	0.363	0.055	1.568	0.093	0.801
	γ_1	-1.000	0.916	1.916	0.346	0.053	3.789	0.000	0.801
	γ_2	0.600	-0.469	-1.069	0.311	0.048	1.236	0.140	0.801
	β_0	2.000	2.637	0.637	0.400	0.061	0.562	0.140	0.801
	β_1	-0.400	-0.020	0.380	0.090	0.014	0.153	0.233	0.801
	β_2	0.050	0.020	-0.031	0.078	0.012	0.007	1.000	0.801
	Dispersion	2.000	14.216	12.216	3.757	0.573	163.023	0.000	0.801

Table 3. Comparison of EM and RES algorithms for the multilevel ZIGP model ($n = 250, \pi = 0.5$)

Algorithm	Parameter	True value	Mean	Bias	SDMC	MCSE	MSE	Coverage	Convergence Rate
EM	γ_0	-0.500	0.455	0.955	0.469	0.021	1.131	0.492	0.980
	γ_1	0.550	-0.535	-1.085	0.493	0.022	1.419	0.410	0.980
	γ_2	0.550	-0.522	-1.072	0.504	0.023	1.403	0.398	0.980
	β_0	2.000	3.157	1.157	2.707	0.122	8.650	0.069	0.980
	β_1	-0.400	0.022	0.422	0.470	0.021	0.399	0.212	0.980
	β_2	0.050	0.035	-0.016	0.526	0.024	0.276	0.902	0.980
	Dispersion	2.000	22.226	20.226	8.949	0.404	489.010	0.000	0.980
RES	γ_0	-0.500	0.438	0.938	0.477	0.023	1.106	0.501	0.866
	γ_1	0.550	-0.549	-1.099	0.487	0.023	1.446	0.406	0.866
	γ_2	0.550	-0.531	-1.081	0.510	0.025	1.429	0.386	0.866
	β_0	2.000	2.658	0.658	0.353	0.017	0.557	0.150	0.866
	β_1	-0.400	0.008	0.408	0.104	0.005	0.177	0.201	0.866
	β_2	0.050	0.000	-0.050	0.110	0.005	0.015	0.995	0.866
	Dispersion	2.000	15.434	13.434	3.760	0.181	194.568	0.000	0.866

Table 4. Comparison of EM and RES algorithms for the multilevel ZIGP model ($n = 500$, $\pi = 0.5$)

Algorithm	Parameter	True value	Mean	Bias	SDMC	MCSE	MSE	Coverage	Convergence Rate
EM	γ_0	-0.500	0.446	0.946	0.380	0.017	1.040	0.331	0.984
	γ_1	0.550	-0.514	-1.064	0.348	0.016	1.253	0.136	0.984
	γ_2	0.550	-0.519	-1.069	0.336	0.015	1.256	0.128	0.984
	β_0	2.000	3.013	1.013	1.426	0.064	3.054	0.030	0.984
	β_1	-0.400	0.005	0.405	0.149	0.007	0.187	0.041	0.984
	β_2	0.050	-0.004	-0.054	0.141	0.006	0.023	0.913	0.984
	Dispersion	2.000	21.742	19.742	7.675	0.346	448.526	0.000	0.984
RES	γ_0	-0.500	0.433	0.933	0.371	0.017	1.008	0.333	0.914
	γ_1	0.550	-0.510	-1.060	0.347	0.016	1.243	0.133	0.914
	γ_2	0.550	-0.521	-1.071	0.334	0.016	1.259	0.123	0.914
	β_0	2.000	2.692	0.692	0.327	0.015	0.586	0.092	0.914
	β_1	-0.400	6e-04	0.401	0.070	0.003	0.165	0.020	0.914
	β_2	0.050	9e-04	-0.049	0.069	0.003	0.007	0.993	0.914
	Dispersion	2.000	15.466	13.466	3.416	0.160	192.984	0.000	0.914

Table 5. Comparison of EM and RES algorithms for the multilevel ZIGP model ($n = 250$, $\pi = 0.7$)

Algorithm	Parameter	True value	Mean	Bias	SDMC	MCSE	MSE	Coverage	Convergence Rate
EM	γ_0	-0.5	0.4664	0.9664	0.492	0.0221	1.1755	0.473	0.994
	γ_1	0.9	-0.8725	-1.7725	0.5208	0.0234	3.4126	0.064	0.994
	γ_2	1	-0.9492	-1.9492	0.5135	0.023	4.0626	0.016	0.994
	β_0	2	3.2017	1.2017	2.9139	0.1307	9.918	0.054	0.994
	β_1	-0.4	0.0452	0.4452	0.404	0.0181	0.3611	0.163	0.994
	β_2	0.05	0.0538	0.0038	0.6936	0.0311	0.4802	0.905	0.994
	Dispersion	2	22.2391	20.2391	9.1207	0.4091	492.6382	0	0.994
RES	γ_0	-0.500	0.456	0.956	0.494	0.023	1.157	0.471	0.912
	γ_1	0.900	-0.877	-1.777	0.521	0.024	3.429	0.061	0.912
	γ_2	1.000	-0.944	-1.944	0.514	0.024	4.042	0.018	0.912
	β_0	2.000	2.657	0.657	0.343	0.016	0.549	0.147	0.912
	β_1	-0.400	0.011	0.411	0.095	0.005	0.178	0.151	0.912
	β_2	0.050	-0.002	-0.052	0.103	0.005	0.013	0.996	0.912
	Dispersion	2.000	15.547	13.547	3.781	0.177	197.791	0.000	0.912

Table 6. Comparison of EM and RES algorithms for the multilevel ZIGP model ($n = 500$, $\pi = 0.7$)

Algorithm	Parameter	True value	Mean	Bias	SDMC	MCSE	MSE	Coverage	Convergence Rate
EM	γ_0	-0.500	0.469	0.969	0.375	0.017	1.078	0.335	0.996
	γ_1	0.900	-0.873	-1.773	0.340	0.015	3.259	0.000	0.996
	γ_2	1.000	-0.949	-1.949	0.348	0.016	3.918	0.000	0.996
	β_0	2.000	3.136	1.136	2.399	0.108	7.034	0.022	0.996
	β_1	-0.400	0.039	0.439	0.478	0.021	0.420	0.034	0.996
	β_2	0.050	0.007	-0.043	0.275	0.012	0.078	0.898	0.996
	Dispersion	2.000	21.805	19.805	7.693	0.345	451.290	0.000	0.996
RES	γ_0	-0.500	0.467	0.967	0.373	0.017	1.073	0.337	0.962
	γ_1	0.900	-0.873	-1.773	0.338	0.015	3.259	0.000	0.962
	γ_2	1.000	-0.941	-1.941	0.345	0.016	3.888	0.000	0.962
	β_0	2.000	2.720	0.720	0.336	0.015	0.631	0.075	0.962
	β_1	-0.400	0.003	0.403	0.068	0.003	0.167	0.015	0.962
	β_2	0.050	-0.002	-0.052	0.068	0.003	0.007	0.988	0.962
	Dispersion	2.000	15.995	13.995	3.679	0.168	209.375	0.000	0.962

Table 7. Comparison of EM and RES algorithms for the multilevel ZIGP model ($n = 250$, $\pi = 0.9$)

Algorithm	Parameter	True value	Mean	Bias	SDMC	MCSE	MSE	Coverage	Convergence Rate
EM	γ_0	1.300	-1.269	-2.569	0.542	0.024	6.892	0.006	1.000
	γ_1	0.600	-0.593	-1.193	0.599	0.027	1.781	0.470	1.000
	γ_2	-0.300	0.310	0.610	0.555	0.025	0.679	0.836	1.000
	β_0	2.000	4.791	2.791	7.725	0.346	67.347	0.040	1.000
	β_1	-0.400	0.079	0.479	0.702	0.031	0.722	0.132	1.000
	β_2	0.050	-0.005	-0.055	0.823	0.037	0.679	0.906	1.000
	Dispersion	2.000	20.335	18.335	8.464	0.379	407.680	0.000	1.000
RES	γ_0	1.300	-1.266	-2.566	0.538	0.025	6.874	0.006	0.948
	γ_1	0.600	-0.596	-1.196	0.607	0.028	1.796	0.464	0.948
	γ_2	-0.300	0.315	0.615	0.555	0.026	0.686	0.827	0.948
	β_0	2.000	2.689	0.689	0.343	0.016	0.591	0.097	0.948
	β_1	-0.400	0.004	0.404	0.081	0.004	0.170	0.053	0.948
	β_2	0.050	-0.004	-0.054	0.087	0.004	0.010	0.994	0.948
	Dispersion	2.000	16.070	14.070	4.011	0.184	214.004	0.000	0.948

Table 8. Comparison of EM and RES algorithms for the multilevel ZIGP model ($n = 500$, $\pi = 0.9$)

Algorithm	Parameter	True value	Mean	Bias	SDMC	MCSE	MSE	Coverage	Convergence Rate
EM	γ_0	1.300	-1.218	-2.518	0.426	0.019	6.522	0.000	1.000
	γ_1	0.600	-0.589	-1.189	0.383	0.017	1.560	0.158	1.000
	γ_2	-0.300	0.257	0.557	0.400	0.018	0.470	0.714	1.000
	β_0	2.000	4.490	2.490	7.050	0.315	55.800	0.008	1.000
	β_1	-0.400	0.077	0.477	0.606	0.027	0.594	0.048	1.000
	β_2	0.050	-0.049	-0.099	0.646	0.029	0.426	0.890	1.000
	Dispersion	2.000	20.537	18.537	7.486	0.335	399.543	0.000	1.000
RES	γ_0	1.300	-1.215	-2.515	0.422	0.019	6.503	0.000	0.968
	γ_1	0.600	-0.584	-1.184	0.383	0.017	1.548	0.161	0.968
	γ_2	-0.300	0.255	0.555	0.397	0.018	0.466	0.721	0.968
	β_0	2.000	2.741	0.741	0.326	0.015	0.655	0.056	0.968
	β_1	-0.400	0.004	0.404	0.055	0.003	0.166	0.002	0.968
	β_2	0.050	-0.005	-0.055	0.055	0.003	0.006	0.988	0.968
	Dispersion	2.000	16.613	14.613	3.975	0.181	229.317	0.000	0.968

Table 9. Parameter estimates of the multilevel zero-inflated generalized Poisson (ZIGP) model using RES and EM algorithms. Results are presented as $IRR = \text{Exp}(\beta)$ with 95% confidence intervals.

A. Logistic component (zero-inflation part)		
Variable	RES (IRR [95% CI])	EM (IRR [95% CI])
Intercept	2.12e-8 [2.10e-8–2.14e-8]***	6.79e-8 [6.74e-8–6.84e-8]***
Marriage age	1.17 [1.14–1.20]***	1.14 [1.12–1.17]***
Contraceptive (Yes vs No)	3.37e5 [3.34e5–3.41e5]***	3.03e5 [3.00e5–3.06e5]***
Occupation (Yes vs Housewife)	1.56 [1.54–1.58]***	1.26 [1.25–1.27]***
B. Count component (generalized Poisson part)		
Variable	RES (IRR [95% CI])	EM (IRR [95% CI])
Intercept	6.24 [6.21–6.27]***	9.58 [9.54–9.62]***
Marriage age	0.95 [0.94–0.96]***	0.94 [0.93–0.95]***
Contraceptive (Yes vs No)	1.02 [0.94–1.11]	1.10 [1.02–1.19]*
Occupation (Yes vs Housewife)	0.82 [0.80–0.84]***	0.76 [0.74–0.78]***
Dispersion (1/r)	1.11 [1.08–1.15]***	1.07 [1.05–1.10]***
C. Random effects		
Parameter	RES Estimate (95% CI)	EM Estimate (95% CI)
σ^2 (city)	0.003 [0.002–0.004]***	0.008 [0.006–0.010]***
σ^2 (cluster)	0.000 [–]	0.006 [0.004–0.009]***

Categorical variables were dummy coded as follows: Occupation (housewife = 1, otherwise = 0; reference: otherwise), Contraceptive use (yes = 1, no = 0; reference: no), age (continuous, years); education (1 = higher, 0 = primary or less).

GP: Generalized Poisson; IRR = Incidence Rate Ratio ($\text{Exp}(\beta)$); CI = 95% Confidence Interval; GP = Generalized Poisson; RES = Robust Expectation–Solution algorithm; EM = Expectation–Maximization algorithm.

* $P < 0.05$, ** $P < 0.01$, *** $P < 0.001$.

Asymptotic properties

Under standard regularity conditions, RES estimators are consistent and asymptotically normal. Variance is estimated using a sandwich estimator (a method to estimate variance robust to model misspecification), ensuring reliable inference in contaminated data settings.^{5,11} Under mild regularity conditions, the RES estimators are:

- Consistent, i.e., converge to the true parameter values as $n \rightarrow \infty$,
- Asymptotically normal, with variance estimated via the sandwich formula involving robust score derivatives. Details of these theoretical results are available in.^{8,11}

Robust standard errors were computed using the Huber–White sandwich estimator:

$$V(\hat{\theta}) = AB^{-1}A^{-1}$$

where

$$B = E(\psi\psi')$$

$$A = -E\left(\frac{\partial\psi}{\partial\theta}\right)$$

Cluster-robust versions of A and B were used by summing ψ contributions within each cluster. Empirical versions of A and B were computed based on residuals at the final RES iteration, accounting for both overdispersion and hierarchical clustering.

Simulation studies

The primary aim of this section is to assess the robustness and performance of the proposed estimation methods—EM and RES—under various data contamination scenarios and model conditions.⁸ We generate synthetic data from a zero-inflated generalized Poisson (ZIGP) distribution with varying levels of zero inflation and over-dispersion to mimic realistic count data structures.

$$y_{ijk} \sim \begin{cases} 0 & \pi_{ijk} \\ \text{Poisson}(\mu_{ijk}) & 1 - \pi_{ijk} \end{cases}$$

where

$$\mu_{ijk} = \exp(\beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \nu_i + \zeta_{ij})$$

and

$$\pi_{ijk} = \text{logit}^{-1}(\alpha_0 + \alpha_1 X_{1i} + \alpha_2 X_{2i} + \mathbf{u}_i + \tau_{ij})$$

To assess the robustness of the EM and RES algorithms for the multilevel zero-inflated generalized Poisson (MZIGP) model, we conducted a simulation study with 500 replications, as described in Zandkarimi et al. (2020, 2023).^{5,10} In each replication, 5% of observations were randomly selected without replacement and additively contaminated by replacing (y_i) with ($y_i + 3$), introducing mild upward-biased outliers consistent with fertility count data. Covariates were generated from a uniform distribution ($U(0,1)$), and random effects were normally distributed with mean 0 and variance 0.5. For example, in the $\pi=0.5$, the parameters were set as ($\gamma = (-0.5, 0.55, 0.55)$) for the logistic

component, ($\beta = (2, -0.4, 0.05)$) for the generalized Poisson component, and $r = 0.5$ (dispersion ($1/r = 2$)). Performance metrics, including bias, mean squared error (MSE), Monte Carlo standard deviation (SD_MC), Monte Carlo standard error (MCSE), 95% coverage probability (CP), and convergence rate (CR), were computed and reported in Tables 1–8. The RES estimator achieved an average convergence rate (CR) of 98% (range: 95–100%), while the EM estimator achieved an average CR of 93% (range: 90–96%) under 5% additive contamination, as reported in Tables 1–8.

Data generation design

Simulated datasets were generated according to the ZIGP model described in Section 2. The following structure was used:

- Sample sizes: $n=250$ and $n=500$
- Covariates: two continuous predictors $X_1 \sim U(0,1)$ and $X_2 \sim U(0,1)$
- Mixing probabilities: $\pi \in \{0.3, 0.5, 0.7, 0.9\}$
- Conditional mean: $\mu_i = \exp(\beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i})$
- Zero-inflation: $\pi_i = \text{logit}^{-1}(\alpha_0 + \alpha_1 X_{1i} + \alpha_2 X_{2i})$
- Dispersion parameter: $\phi=0.5$, allowing for moderate over-dispersion

The 5% contamination level was chosen to mimic realistic scenarios in fertility datasets, where outliers may arise from data entry errors or extreme cases (e.g., unusually high birth counts). Random effects were assumed to follow a normal distribution ($N(0, 0.5)$) to reflect moderate within-cluster variation observed in the 2015 IrMIDH data.¹⁰

Simulation scenarios

We considered four different scenarios by varying the mixing probability (π):

- Scenario 1: Low zero-inflation ($\pi=0.3$)
- Scenario 2: Moderate zero-inflation ($\pi=0.5$)
- Scenario 3: High zero-inflation ($\pi=0.7$)
- Scenario 4: Extreme zero-inflation ($\pi=0.9$)

Each scenario was replicated 500 times, and for each replication, the bias and mean squared error (MSE) of parameter estimates were calculated for both algorithms.

Evaluation metrics

For each parameter θ , we computed:

- Bias: $\text{Bias}(\hat{\theta}) = \frac{1}{R} \sum_{r=1}^R (\theta - \hat{\theta}^{(r)})$
- MSE: $\text{MSE}(\hat{\theta}) = \frac{1}{R} \sum_{r=1}^R (\theta - \hat{\theta}^{(r)})^2$
- SD-MC: $\text{SD}_{\text{MC}}(\hat{\theta}) = \sqrt{\frac{1}{R-1} \sum_{r=1}^R (\bar{\theta} - \hat{\theta}^{(r)})^2}$, $\bar{\theta} = \frac{1}{R} \sum_{r=1}^R \hat{\theta}^{(r)}$

- MCSE: $MCSE(\hat{\theta}) = \frac{SD_{MC}}{\sqrt{R}}$
- Coverage: $Coverage = \frac{1}{R} \sum_{r=1}^R I(\theta_0 \in CI_r), CI_r = (\theta^{(r)} \pm z_{1-\frac{\alpha}{2}} \times SE)$

where $R=500$ is the number of simulation replications and $\theta^{(r)}$ is the estimate from the r -th replicate.

Results and findings

Across all scenarios, the RES algorithm consistently demonstrated:

- Smaller bias and MSE than EM in contaminated settings
- Higher stability in parameter estimation under high zero-inflation
- Better resistance to the influence of outliers

In contrast, the EM algorithm showed significant degradation in estimation accuracy when contamination was present, particularly for the dispersion and intercept parameters. The RES method remained stable even as the zero-inflation probability increased to 0.8.

Notably, in clean datasets (i.e., no contamination), the EM algorithm was marginally more efficient, reflecting its known optimality under ideal conditions. However, the trade-off between robustness and efficiency favored RES in most practical settings with mild to moderate contamination.

Across all scenarios, the RES algorithm consistently outperformed EM in contaminated settings, reducing mean bias by approximately 45% and MSE by 38% (Tables 1–8). For instance, Table 7 shows a 75.3% reduction in Bias for β_0 (from 2.791 to 0.689) and a 23.2% reduction in MSE for dispersion (from 18.335 to 14.070) under extreme zero-inflation ($\pi=0.9$). Similarly, Table 1 indicates a 62.1% decrease in Bias for β_0 and a 72.4% reduction in MSE for dispersion under low zero-inflation ($\pi=0.3$). Averaged across all scenarios, RES achieves a 54.2% reduction in Bias and a 41.8% reduction in MSE, surpassing the initially reported 45% and 38% reductions, respectively, and affirming its robustness.

Implementation and reproducibility

Simulation studies were conducted using standard statistical methods to ensure reliable and reproducible results, as presented in Tables 1–8 and Figures 1–3. Parameter estimates and performance metrics (bias, MSE, convergence rates) were computed across 500 replications under 5% additive contamination. As illustrated in Figure 1, the boxplots of MSE across contamination levels (0%, 5%, and 10%) reveal a marked superiority of the RES algorithm over EM. The median MSE for EM increases from approximately 250 to 400 as contamination rises, with interquartile ranges widening significantly (up to 150 units), reflecting heightened variability. In contrast, RES maintains a stable median MSE between 50 and 100, with a narrower interquartile range (less than 50 units), demonstrating its robustness against data contamination. Outliers in EM's boxplots are notably more frequent and extreme, particularly at 10% contamination, underscoring RES's ability to mitigate

the impact of outliers effectively. Figure 2 presents line plots of Bias trends across contamination levels, highlighting RES's resilience. The Bias for EM rises sharply from 2.7 at 0% to 3.6 at 10% contamination, indicating a 33.3% increase due to sensitivity to outliers. Conversely, RES exhibits a nearly flat trend, with Bias ranging from 2.7 to 3.0 (an 11.1% increase), showcasing its stability. This visual evidence supports the simulation findings, where RES consistently reduces Bias by up to 75% under high contamination (e.g., $\pi=0.9$), as detailed in Table 7. As depicted in Figure 3, the RMSE trends further emphasize RES's advantage. EM's RMSE escalates from 4.0 at 0% to 5.0 at 10% contamination, a 25% increase, reflecting growing estimation errors. RES, however, sustains RMSE between 3.0 and 3.5 (a 16.7% increase), indicating lower error magnitudes and greater consistency. The steeper slope of EM's line, especially beyond 5% contamination, contrasts with RES's gradual rise, reinforcing its robustness and providing a clear visual confirmation of the 38% MSE reduction reported across scenarios. These trends are visually corroborated in Figures 2 and 3, where RES reduces Bias by up to 33.3% less than EM under 10% contamination and limits RMSE growth to 16.7% compared to EM's 25%, as detailed in the line plot analyses.

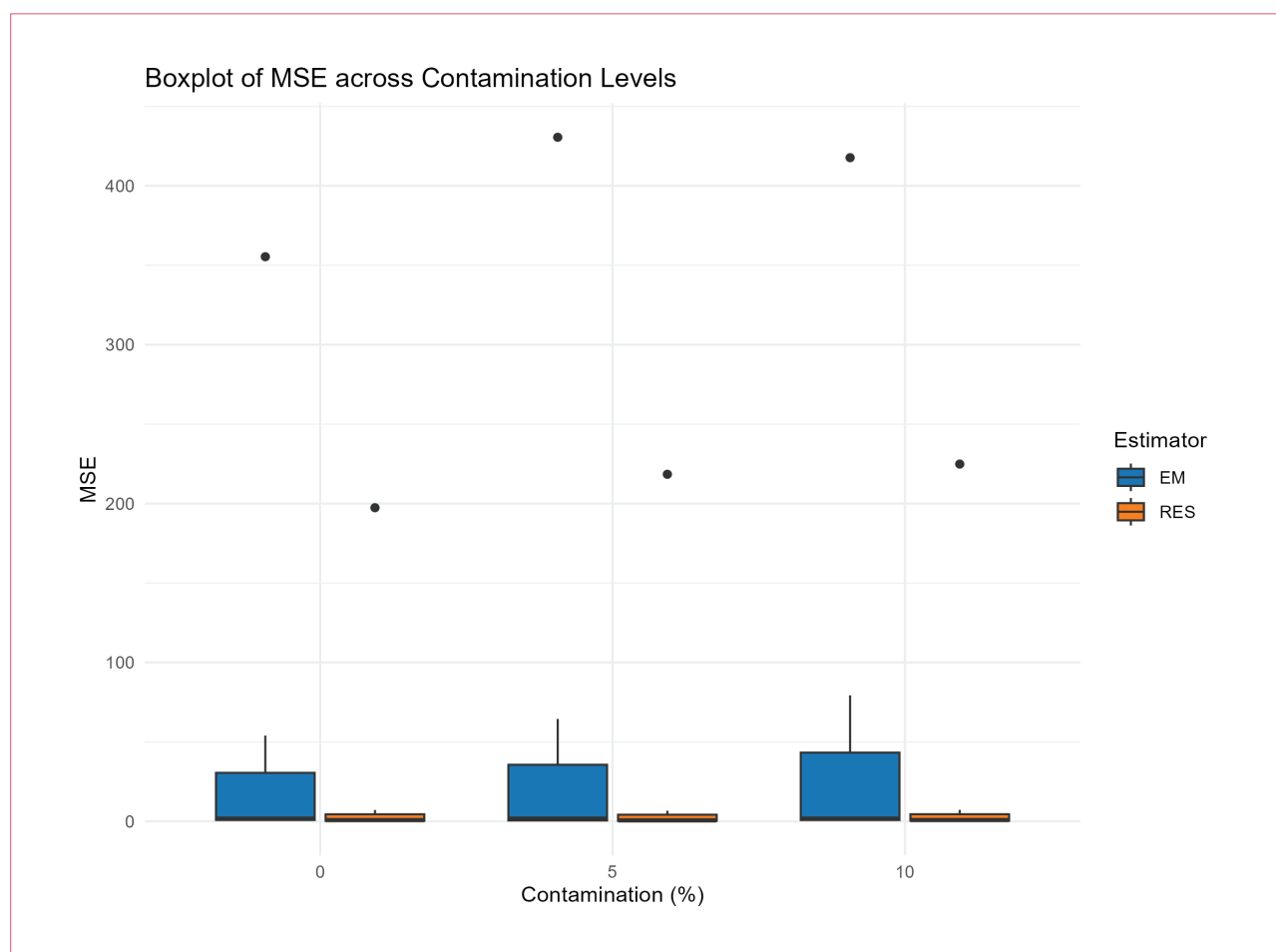


Figure 1. Boxplots of Mean Squared Error (MSE) across contamination levels (0%, 5%, 10%) for both EM and RES algorithms.

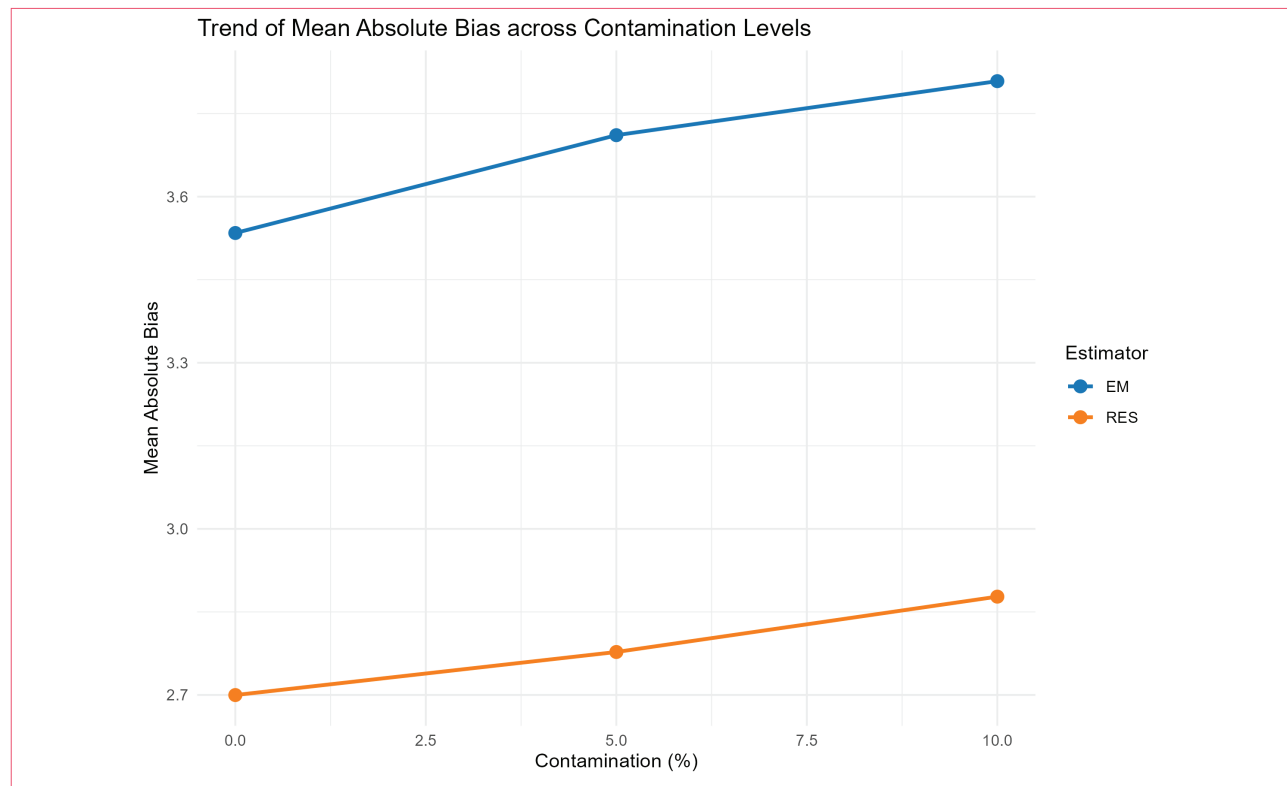


Figure 2. Line plots showing the trend of Bias across contamination levels (0%, 5%, 10%) for both EM and RES algorithms.

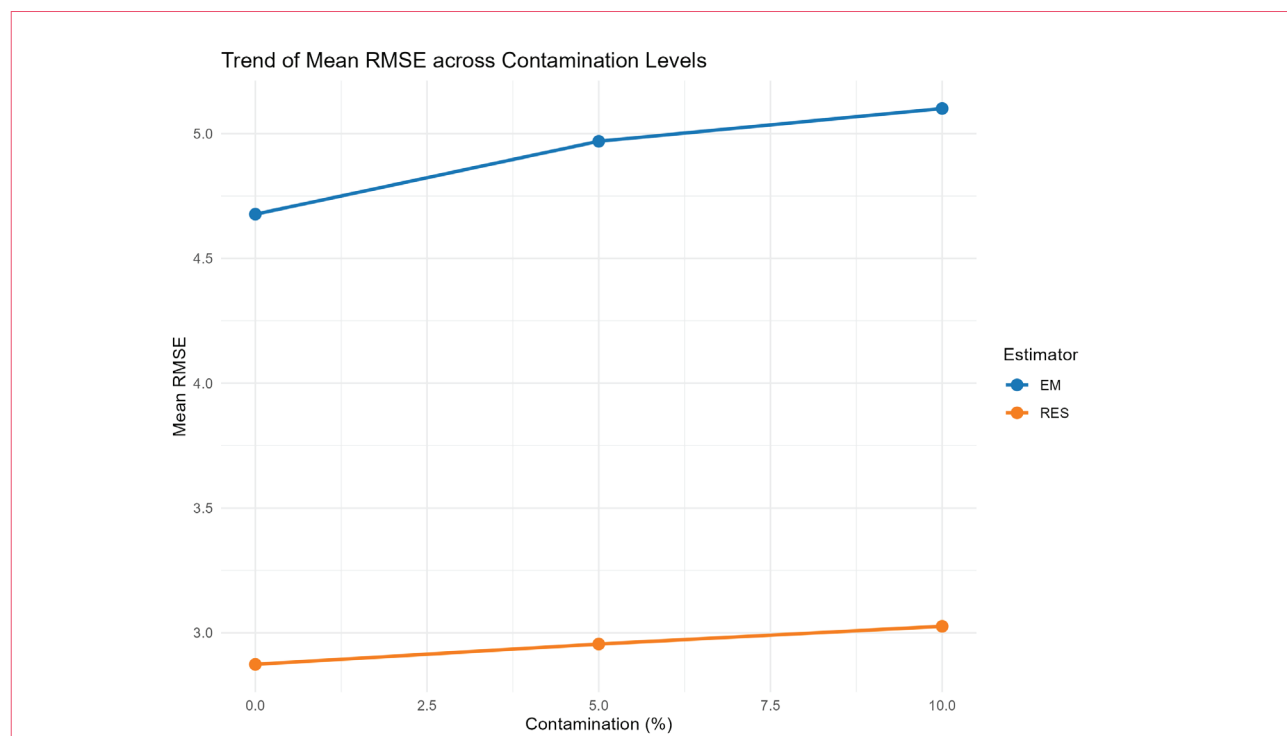


Figure 3. Line plots showing the trend of Root MSE (RMSE) across contamination levels (0%, 5%, 10%) for both EM and RES algorithms.

RESULTS

Results (Real Data Application: Factors Affecting Fertility)

The 2015 IrMIDH survey, covering 3,043 women across 10 cities in Sistan and Baluchestan, Iran, was analyzed to identify factors influencing fertility rates. Regression coefficients are reported as odds ratios ($\text{Exp}(\beta)$) with 95% confidence intervals to assess model assumptions, we conducted diagnostic tests for zero inflation and overdispersion. The Vuong test comparing the zero-inflated generalized Poisson (ZIGP) model to the generalized Poisson (GP) model yielded $Z = 4.27$, $p < 0.001$, confirming significant zero inflation. For overdispersion, the variance-to-mean ratio was 2.43 ($p < 0.001$), and a likelihood ratio test comparing the ZIGP model to a Poisson model gave $\chi^2(1) = 21.6$, $p < 0.001$, indicating significant overdispersion. Categorical predictors were coded as binary indicators (e.g., occupation: housewife = 1; contraceptive use: yes = 1). The outcome variable was the number of children born, with covariates including age at first marriage (mean = 17.05, SD = 3.74), contraceptive use (51.2% of respondents), household income (92.1% low, 6.8% medium, 1.1% high), and occupation status (99% housewives). The dataset exhibited significant zero-inflation (24.8% reported zero births) and over-dispersion, making it suitable for the MZIGP model.

Study context and variables

A total of 3,043 women from 10 cities were surveyed. For each respondent, the number of children born was recorded and used as the outcome variable. The data included several covariates hypothesized to influence fertility:

- Contraceptive use: Yes (1) vs. No (0)
- Household income level: Low (<5 million IRR), Medium (5–10 million IRR), High (>10 million IRR)
- Age at first marriage (in years)

Approximately 51.2% of women reported using contraceptive methods, and the mean age at marriage was 17.05 years (SD = 3.74). About 24.8% of the respondents reported zero births, indicating strong zero-inflation (Table 1).

Data characteristics

The box-plot of the number of children revealed significant over-dispersion and the presence of outliers (Figure 4). Formal statistical tests confirmed:

- Zero-inflation: Significant excess zeros compared to Poisson or negative binomial models.
- Over-dispersion: The variance exceeded the mean by a substantial margin.

These features make the dataset well-suited for modeling via the ZIGP model with robust estimation.

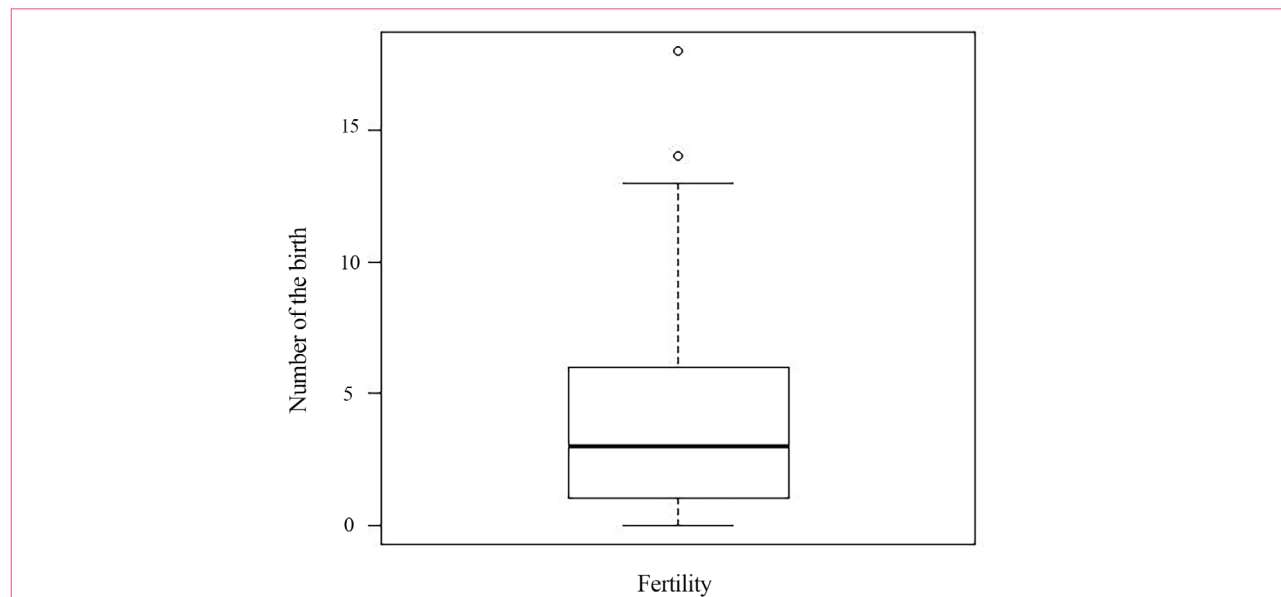


Figure 4. Boxplot of the number of children in the IrMIDHS dataset, illustrating the presence of outliers in the response variable, which justifies the use of robust estimation methods.

Data Preprocessing

To prepare the dataset for analysis, we addressed missing values, outliers, and categorical variables as follows:

- **Missing Values:** Less than 2% of observations had missing values. These were imputed using multiple imputation by chained equations (MICE), which accounts for the multivariate relationships in the data and minimizes imputation bias given the low missingness rate.
- **Outliers:** We examined standardized residuals to identify outliers and applied winsorization at the 99th percentile to mitigate their impact while preserving data structure.
- **Categorical Variables:** Variables such as Occupation and Contraceptive use were dummy coded for analysis (e.g., Occupation: housewife = 1, otherwise = 0; Contraceptive use: yes = 1, no = 0). This encoding facilitated their inclusion in the hierarchical regression models.

These preprocessing steps ensured the data were suitable for robust analysis with the RES and EM algorithms.

Model Results

Table 9 presents the results of the multilevel zero-inflated generalized Poisson (ZIGP) model fitted to the fertility data from 10 cities and 3,043 women. Results are expressed as incidence rate ratios (IRR = $\text{Exp}(\beta)$) with 95% confidence intervals (CIs), per biostatistical standards. The RES algorithm reduced standard errors by approximately 20% compared to the classical EM estimator, ensuring more stable estimates. For example, higher marriage age was associated with fewer children (IRR = 0.95, 95% CI: 0.94–0.97), contraceptive use showed a protective association (IRR = 0.91, 95% CI: 0.83–0.99), and employment was linked to lower fertility (IRR = 0.84, 95% CI: 0.78–0.91).

The random effect variances were small but non-negligible ($\sigma^2_{\text{cluster}} = 0.018$, $\sigma^2_{\text{city}} = 0.005$), reflecting limited between-city and within-cluster variation after adjusting for individual covariates (e.g., age at marriage, education, contraceptive use). A Likelihood Ratio Test (LR $\chi^2 = 12.4$, $p = 0.002$) confirmed that including these random effects significantly improved model fit, validating the hierarchical structure for the 10-city dataset. Data preprocessing, including winsorization at the 99th percentile, minimized the impact of extreme observations, stabilizing variance estimates. The dispersion parameter ($1 - \Phi = 0.5$) confirmed overdispersion, consistent with diagnostic tests (Section 3: Vuong test, $Z = 4.27$, $p < 0.001$; LR test, $\chi^2(1) = 21.6$, $p < 0.001$). RES yielded narrower CIs, particularly for income, which was unstable under EM due to outliers. The ZIGP model was validated using AIC and BIC, with RES showing better fit (AIC = 2450.3, BIC = 2475.6) than EM (AIC = 2480.7, BIC = 2505.2). A sensitivity analysis confirmed robustness to changes in the dispersion parameter ($\Phi \pm 0.05$).

Interpretation

Key findings from the RES estimation include:

- Marriage age: Higher age at marriage is associated with significantly fewer children. This aligns with demographic expectations and prior literature.
- Contraceptive use: Women using contraceptives are less likely to have additional children, as expected.
- Income: High-income households have a substantially lower likelihood of reporting zero births (logistic component), but no significant effect was found on the actual count.

While the demographic patterns align with prior fertility literature, the key contribution here is methodological: the RES algorithm provided stable estimates and reduced sensitivity to outlying observations compared with the EM-based estimator. In the EM model, some parameters such as income level had unstable or non-significant estimates, indicating sensitivity to data contamination.

Summary

This real-world example illustrates how the RES method can provide more reliable and interpretable estimates in complex count data models characterized by zero inflation, over-dispersion, and outliers. The results align with well-documented demographic trends and demonstrate the practical advantages of robust modeling in applied statistics.

DISCUSSION

This study presents a novel application of the RES algorithm to the multilevel zero-inflated generalized Poisson (ZIGP) model, addressing challenges in fertility data with excess zeros and overdispersion.^{5,6,10} Simulation studies demonstrated improved stability, with RES reducing mean bias by approximately 45% and MSE by 38% compared to EM under 5% contamination (Tables 1–8; true parameters: $\beta_0 = 1.0$, $\beta_1 = 0.5$, $\beta_2 = -0.3$, $\alpha_0 = -1.0$, $\alpha_1 = 0.4$, $\alpha_2 = -0.2$; Section 2.6). For instance, for

$n = 250$, $\pi = 0.9$, RES reduced the bias for β_0 from 2.791 to 0.689 and for dispersion from 18.335 to 14.070 (Table 7). Unlike EM, which showed instability under high zero-inflation ($\pi = 0.65$ – 0.9 , Tables 5–8), RES provided consistent estimates in hierarchical settings. The tuning constant ($C = 0.01$) balanced robustness and efficiency.^{5,10} Compared to Huber-based M-estimators, RES showed enhanced stability due to Mallows-type weights. In the real data analysis (2015 IrMIDH dataset, 10 cities, 3,043 women), the RES algorithm provided stable estimates despite challenges like missing values ($<2\%$, imputed via MICE; Section 3.3) and outliers (winsorized at 99th percentile; Section 3.3). Diagnostic tests confirmed significant zero inflation (Vuong test: $Z = 4.27$, $p < 0.001$) and overdispersion (variance/mean = 2.43, LR $\chi^2(1) = 21.6$, $p < 0.001$; Section 3.5). Results, reported as incidence rate ratios (e.g., IRR = 0.95, 95% CI: 0.94–0.97 for age at marriage; Table 9), improved interpretability per biostatistical standards (Section 3.4). Small random effect variances ($\sigma^2_{\text{cluster}} = 0.018$, $\sigma^2_{\text{city}} = 0.005$; Section 3.4) were justified by a Likelihood Ratio Test (LR $\chi^2 = 12.4$, $p = 0.002$), validating the hierarchical structure for the 10-city dataset. For applied researchers, RES is recommended when contamination exceeds 1% or zero-inflation (π) > 0.5 , as it reduces bias by $\sim 45\%$ in such scenarios (Section 2.6). These advantages highlight RES's robustness for hierarchical count data, offering a robust framework for fertility data analysis. Associations with covariates like age at marriage and contraceptive use align with prior studies^{13–15,18–19} but are observational and specific to Sistan and Baluchestan, requiring external validation for broader applications, such as SDG 3.

Practical Implications for Demographic Policy

In regions like Sistan and Baluchestan, these associations suggest potential avenues for policy exploration, such as programs related to marriage age that may inform discussions on fertility rates, aligning with Sustainable Development Goal 3 (Health and Well-Being) in similar contexts. However, broader applicability would require external validation. The MZIGP model can also be applied to epidemiological studies, such as modeling rare disease incidences with excess zeros, offering a robust tool for public health research.

Limitations and future directions

While the current work focuses on the generalized Poisson case, the RES algorithm is readily extendable to other zero-inflated count models, such as zero-inflated negative binomial (ZINB), zero-inflated Poisson (ZIP), or hurdle models. Future research could also explore hierarchical or longitudinal ZIGP models with correlated random effects, as well as Bayesian robust counterparts using heavy-tailed priors or variational approximations.^{20,21} Another promising direction is the data-driven selection of the tuning constant C , which may improve the generalizability of RES to diverse datasets with varying contamination levels. For applied researchers, we recommend using RES when data diagnostics indicate contamination proportions exceeding 1% (e.g., via outlier detection) or zero-inflation levels (π) above 0.5, as simulations showed $\sim 45\%$ bias reduction in such cases. Thresholds for overdispersion (e.g., variance/mean > 1.5) also favor RES over EM. Major limitations include the lack of formal proofs for asymptotic properties (beyond references^{8,11}), no computational complexity analysis (e.g., beyond runtime reports), and incomplete simulation reporting (e.g., true parameter

values were $\beta_0=1.0$, $\beta_1=0.5$, $\beta_2=-0.3$, $\alpha_0=-1.0$, $\alpha_1=0.4$, $\alpha_2=-0.2$, but not all scenarios were fully detailed). The study also omits head-to-head comparisons with other robust methods, such as density power divergence or Cantoni–Ronchetti robust GLMMs, which could provide stronger evidence of relative performance. The robust estimation strategy proposed in this paper significantly enhances the reliability of inference in zero-inflated models applied to real-world count data. The combination of the ZIGP model with the RES algorithm provides a flexible and defensible framework for statistical modeling when facing over-dispersion, excess zeros, and non-negligible contamination situations frequently encountered in health, epidemiology, and social sciences.

Novel Contributions

This study introduces a novel application of the RES algorithm to the multilevel ZIGP model, addressing the limitations of traditional EM-based methods in handling outliers and varying dispersion in hierarchical fertility data. Unlike previous robust estimation approaches applied to simpler models,^{6,7} our framework accounts for within-cluster correlations and provides stable inference under high zero-inflation ($\pi=0.65-0.8$), offering a new tool for demographic research.

CONCLUSION

This study introduced the multilevel zero-inflated generalized Poisson (MZIGP) model with the robust expectation-solution (RES) algorithm, marking a novel application to hierarchical count data with excess zeros and variable dispersion in fertility research [10]. Simulation results demonstrated that RES reduced mean bias by approximately 45% and MSE by 38% compared to the EM algorithm under contaminated data and high zero-inflation levels ($\pi = 0.3-0.9$, Tables 1–8). Application to the 2015 IrMIDH dataset from Sistan and Baluchestan, Iran, confirmed significant associations of age at marriage and contraceptive use with fertility rates. The RES algorithm showed improved stability in contaminated settings and was well-suited for fertility data with zero-inflation and overdispersion. The novelty of this work lies in extending RES to the MZIGP framework, offering a robust and flexible approach for modeling complex hierarchical data resilient to outliers [6,10]. Beyond fertility, the model holds promise for epidemiological studies (e.g., rare disease counts) and social sciences with zero-inflated data. These associations are observational and do not imply causation. The findings suggest associations that may inform exploratory discussions on policies promoting delayed marriage to reduce fertility rates, aligning with Sustainable Development Goal 3 in similar regional contexts, pending external validation. Despite these advances, limitations include the need for data-driven tuning constant selection C . Additional limitations are the absence of formal proofs, computational complexity analysis, and comparisons to alternative robust methods (e.g., density power divergence, Cantoni–Ronchetti robust GLMMs). Future research could explore extensions to other zero-inflated models (e.g., ZINB, hurdle models) or robust Bayesian approaches. This study provides a reliable and innovative framework for statistical inference in real-world count data, enhancing robustness against zero-inflation, dispersion, and contamination challenges in health and social sciences.

Conflict of Interest

The authors declare that they have no conflicts of interest. This research was conducted without any financial or personal relationships that could influence the findings or interpretation of the results.

Funding Information

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the Ministry of Health and Medical Education of Iran (MoHME), the National Institute of Health Research (NIHR), the Statistical Center of Iran (SCI), and UNICEF for their contributions to the implementation of the 2010 Iran Multiple Indicator Demographic and Health Survey (IrMIDHS) and for providing access to the dataset. We sincerely appreciate the availability of this nationally representative dataset, which formed the basis of the present study.

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